Sensitivity Analysis of Potential Wake Encounters to Stochastic Flight-Track Parameters

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***Abstract*—Many NextGen technologies and concepts, when implemented, will change the underlying stochastic nature of flight tracks. This paper investigates how changes in the stochastic variability of flight tracks have the potential to impact the probability of potential wake encounters. We investigate these relationships for a single runway and for parallel runways. The results show that the mean and standard deviation of separation, and the vertical standard deviation are the key parameters affecting potential wake encounters. In the parallel-runway scenario, effects of crosswind and runway spacing are studied. A full factorial design is conducted to estimate the impact of parameters and their interactions, with dominating factors identified. In some cases, results can be different even for the same parameter when different circulation thresholds are used for defining wakes. Outputs from different wake models (TDAWP vs. D2P) are compared. The overall objective is to assess the sensitivity of these results to parameters of the stochastic input distributions.**

Keywords—wake vortices; simulation; sensitivy analysis

I. INTRODUCTION

The objective of this paper is to demonstrate how changes in the stochastic variability of flight tracks have the potential to impact separation constraints related to wake vortices. This analysis is carried out via a sensitivity analysis using existing wake-vortex models.

Wake-vortex separation requirements form a key capacity constraint at airports. These requirements ensure that there is sufficient distance between successive aircraft so that a trailing aircraft does not encounter the wake vortex generated by a leading aircraft. Separation requirements must be designed with the variability of the system in mind and thus must be conservative to handle worst-case scenarios. Because aircraft do not all fly the exact same path, separation requirements must ensure safe separation for *all* flights – not just an "average" or "typical" flight.

Many NextGen technologies, when implemented, will change the underlying stochastic nature of flight tracks. For example, required navigation performance (RNP) reduces the variability in the *lateral* dimension. Trajectory-based operations (TBO) seek to provide tighter bounds on the time separation between two successive aircraft. Historical estimates for the standard deviation of time separation at the threshold are within the range of 15-20 seconds under periods of high demand (Lebron 1987; Boswell 1993; Ballin and Erzberger 1996; Andrews and Robinson 2001; Shortle, Zhang, and Wang, 2010). NextGen may reduce the variability in time separation of arriving aircraft at the threshold.

Such changes in the variability of flight tracks have the potential to impact future NextGen separation requirements associated with wake vortices. For example, a reduction in the variability of the time separation reduces the fraction of landing pairs that have short separation times. Because of this, it may be possible to reduce the *average* separation time while maintaining the same level of safety.

This paper conducts a sensitivity analysis of various stochastic parameters associated with arrival flight tracks. Probability density functions associated with these tracks are integrated with wake models. By varying the input stochastic parameters, it is possible to see which parameters have the largest potential impact on wake-vortex constraints.

We use two different wake-vortex evolution models – a wake vortex model developed at NASA, TDAWP (TASS Driven Algorithm for Wake Prediction) (Proctor, Hamilton, and Switzer 2006) as well a model developed at DLR, D2P (Deterministic 2-Phase) (Holzapfel 2003). The models predict the strength and location of a wake vortex based on various input parameters, such as an aircraft's airspeed, weight, and wingspan, and various atmospheric parameters such as turbulence and stratification. Aircraft locations are simulated using hypothetical probability density functions (PDFs) and the wake locations are simulated using either the TDAWP or D2P model. By integrating these two components together, it is possible to estimate the probability of a potential wake encounter.

We specifically investigate the impact of the following stochastic flight-track parameters: separation time (average and standard deviation), lateral position (standard deviation), vertical position (standard deviation), airspeed (mean and standard deviation), crosswind speed (mean and standard

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deviation), runway separation, eddy dissipation rate, and Brunt-Vaisala frequency.

II. Methodology

Monte-Carlo simulation is used to estimate the probability of a potential wake encounter for landing aircraft. We consider both single-runway and parallel-runway scenarios. In these scenarios, aircraft fly along pre-determined routes. Thus, the problem can be analyzed by investigating aircraft and wake behavior at specific cross-sectional areas along the approach path. In other words, rather than considering full flight paths in three-dimensional space, we consider two-dimensional cross-sectional areas at fixed points along the approach path. For example, Figure 1 shows a cross-sectional area at a specific distance from the threshold. The probabilistic nature of these crossing points can be specified by probability density functions (PDFs) in the lateral and vertical dimensions. Wake behavior is simulated only in the two-dimensional slice. In this paper, the location of the cross-section is selected to be 3 nm from the threshold, which corresponds to an altitude of approximately 950 above ground level, assuming a 3-degree glideslope. Only out-of-ground wake effects are considered.

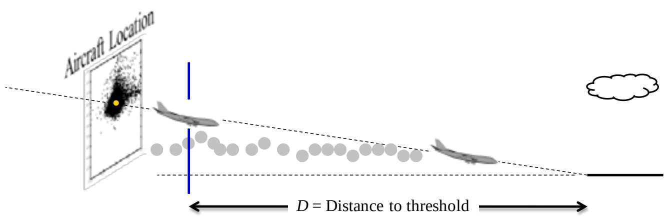

Figure 1. Two-dimensional cross-section on arrival path

Figure 2 shows a notional description of the overall methodology. The methodology is adapted from Shortle and Jeddi (2007) and Shortle (2007). The basic idea is to combine two basic pieces of information: (1) the probabilistic distribution of the locations, speeds, and separations of aircraft, and (2) the probabilistic distribution of wake locations and strengths. All analysis is carried out with respect to a fixed two-dimensional cross-section at some specific distance from the threshold.

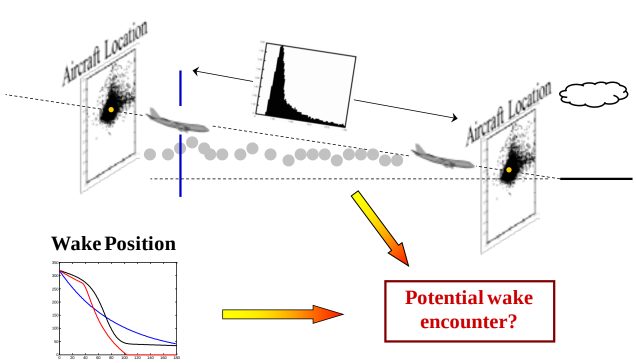

Figure 2. Overall methodology

Variables associated with the aircraft (lateral position, vertical position, etc.) are modeled as random variables with associated PDFs. Baseline parameters of the PDFs are chosen roughly to correspond to observed flight tracks 3 nm from the threshold (e.g., Hall and Soares 2008, Zhang et al. 2010). Since the main contribution of this work is a *sensitivity* analysis, the precise values of these parameters are not critical. The emphasis is rather on observing how *changes* in the parameters affect the overall probability of a potential wake encounter. In Shortle (2007) and Shortle and Jeddi (2007), the aircraft tracks came from actual flight-track data.

For modeling wakes, we have implemented TDAWP from the publicly available description in Proctor, Hamilton, and Switzer (2006). The algorithm is implemented in C++. As a verification check, we have been able to reproduce key figures given in Proctor, Hamilton, and Switzer (2006). Future work will incorporate an updated version of TDAWP into the analysis. We have also implemented a reduced version of the D2P model from the publicly available description in Holzapfel (2003). In the GMU versions of both models, wakes are assumed to descend vertically and the lateral positions of the wakes vary precisely with crosswind. Other existing wake models include APA (AVOSS Prediction Algorithm; Robins and Delisi 2003) and VIPER (Vortex Algorithm Including Parameterized Entrainment Results).

Table I shows the input parameters for the GMU-implemented versions of TDAWP and D2P, along with sample values. The parameters refer to the wake-generating (leading) aircraft. *In this analysis, the mass M and wingspan B are fixed with values as shown in the table (corresponding to a B737-900).* In other words, the analysis in this paper assumes a uniform fleet mix.

TABLE I. Input to GMU implementations of TDAWP and D2P

Input	Sample Value	Definition
V	Variable	Velocity of the leading aircraft
M	66,360 kg	Aircraft mass (B737-900)
B	34.32 m	Wingspan (B737-600/700/800/900)
h	1,000 ft	Altitude of airplane above ground
e	Variable	Eddy dissipation rate
N	Variable	Brunt-Vaisala frequency
r	1.2 kg/m^3	Air density (near ground)

The outputs of the wake models are:

- Vertical position of wake-vortex center as a function of time (relative to location at the point of generation)
- Lateral position of wake-vortex center as a function of time (relative to location at the point of generation)
- Circulation strength as a function of time

This analysis considers both a single-runway scenario (Figure 1) and a parallel-runway scenario. The basic approach for the single-runway scenario consists of the following steps:

1. Generate random variables associated with the leading aircraft as it crosses the cross-sectional area (at a specific distance from the threshold). These variables include lateral position, vertical position, and air speed.
2. Generate the random time separation t until the next aircraft crosses the same cross-sectional area. For a fixed time separation, the distance between the leading and following aircraft varies, depending on the speed of the trailing aircraft.
3. Simulate the wake behavior from the leading aircraft in the cross-sectional area (using either TDAWP or D2P) up to a wake age of t seconds. The resulting output gives the wake state at the time that the trailing aircraft crosses the cross-sectional area.
4. Generate random variables associated with the trailing aircraft as it crosses the cross-sectional area. These include lateral position, vertical position, and air speed.
5. Determine if a potential wake encounter has occurred based on the location and strength of the wake from the leading aircraft and the position of the trailing aircraft within the cross-sectional area (see the next subsection for the definition of a potential wake encounter).
6. Repeat this process where the trailing aircraft becomes the leading aircraft in the next pair.

For the parallel-runway scenario, the methodology is similar, but the analysis is complicated by the possibility that a trailing aircraft may encounter a wake both from the leading aircraft on the same track as well as an aircraft on the parallel track. As before, the analysis is conducted at a specific cross-sectional area of the approach path. The location of each aircraft as it crosses the area is simulated as well as the wake behavior. The main difference is that the parallel-runway model must do extra accounting to keep track of which aircraft on the opposite runway is "just ahead" of the "trailing" aircraft on the current runway. For completeness, the two most recent arrivals on the opposite runway are considered as potential candidates for wake events relative to the current aircraft.

A. Definition of potential wake encounters

Figure 3 defines the potential wake-encounter zone. We use the word *potential*, because the zone represents an area near the wake, but does not describe exactly where the wake is. *An aircraft that is inside the zone may be near a wake but does not necessarily encounter a wake.* Furthermore, all of the analysis in this paper is conducted using models and hypothetical PDFs, not real data, so this is not an evaluation of the current system.

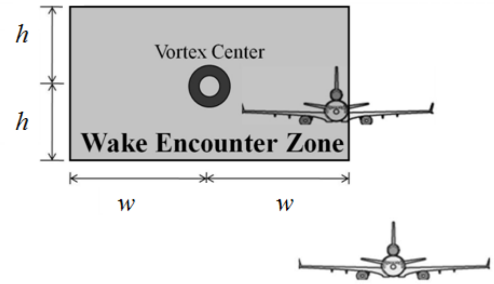

Figure 3. Wake event zones

More specifically, a ***potential wake encounter*** occurs if all three of the following conditions are satisfied simultaneously (at the time when the trailing aircraft is at the specified along-track position):

- The lateral position of the wake from the leading aircraft and the lateral position of the trailing aircraft are within $\pm w$ of each other,
- The vertical position of the wake from the leading aircraft and the lateral position of the trailing aircraft are within $\pm h$ of each other,
- The circulation of the wake has not decayed below a specified threshold (e.g., 75 m^2/s)

The last criterion implies that some potential encounters are not counted in the analysis – namely those in which the trailing aircraft enters the wake zone, but the circulation strength of the encountered wake is below the specified threshold. A circulation threshold of 75 m^2/s corresponds roughly to background atmospheric turbulence levels. This paper also considers several higher threshold values, such as 150 m^2/s and 225 m^2/s.

B. Parameters

Figure 4 shows a graphical illustration of some of the parameter values used in this analysis and their impact on the PDFs associated with the aircraft behavior. *All PDFs in this analysis are assumed to be normal distributions.* (In practice, the time-separation distribution is skewed with a longer right-tail; however, the left side of the distribution, which is what matters in this analysis, does look roughly normal. See Shortle et al. 2010). The following parameters are used in the sensitivity analysis.

- Separation mean (*sepMean*): the mean of the separation time between a trailing aircraft and its leading aircraft (along the same arrival track),
- Separation standard deviation (*sepStd*): the standard deviation of separation time between a trailing aircraft and its leading aircraft,
- Lateral standard deviation (*yStd*): the standard deviation of lateral position of an aircraft at a specific point along the approach path,
- Vertical standard deviation (*zStd*): the standard deviation of vertical position of an aircraft at a specific point along the approach path,

- Velocity mean (*vMean*): the mean airspeed of an aircraft at a specific point along the approach path.
- Velocity standard deviation (*vStd*): the standard deviation of an aircraft's airspeed at a specific point along the approach path,
- Eddy dissipation rate (*e*), an atmospheric parameter representing turbulence,
- Brunt-Vaisala frequency (*N*), an atmospheric parameter representing in some sense the vertical pressure gradient,
- Wind mean (*windMean*): average crosswind speed,
- Runway separation (*runwaysep*): Lateral separation of parallel runways (used only in parallel-runway case),

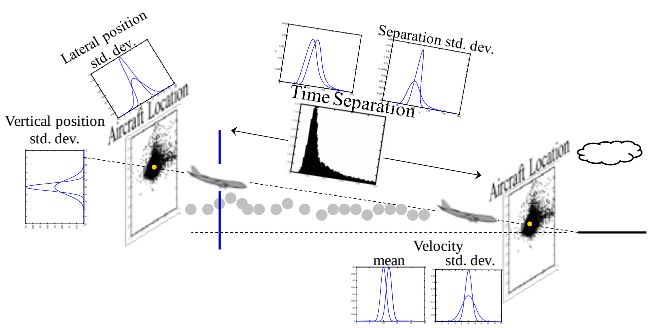

Figure 4. Illustration of PDF parameter values sensitivity analysis

III. SAMPLE RESULTS: SINGLE-RUNWAY MODEL

Two experimental designs are conducted in this paper: Varying each parameter one at a time, and a full-factorial experiment on all parameters.

A. *Variation of single parameters, one at a time*

Starting from the baseline, each parameter is varied individually from its low value to its high value, with other parameters remaining at their baseline values (Table II). The high values for lateral standard deviation and vertical standard deviation correspond roughly to flight-track observations at 3 nm from the threshold in IMC (e.g., Hall and Soares 2008, Zhang et al. 2010). The high values for separation mean and separation standard deviation correspond roughly to observed separations at the threshold (e.g., Shortle et al. 2010, Lebron 1987), which may be close to values observed at 3 nm from the threshold. The values for velocity standard deviation may be high relative to current operations at 3 nm from the threshold. Again, the values are meant to represent *hypothetical* changes in the probability distributions and may not be realizable in actual operations (e.g., 70-second separations). The purpose is to test for key effects resulting from changes (and in particular reductions) in the underlying stochastic parameters, not to evaluate the current system.

For each set of parameter values, 10 replications are conducted, where each replication consists of 100,000 simulated flights. Based on the output of the 10 replications, 95% confidence intervals are calculated. The confidence intervals are calculated based on the assumption that the sample averages follow a normal distribution. (This is typically a reasonable assumption when the number of observed instances is not too small, but for extremely rare events, this assumption may not be that accurate.)

TABLE II. VARYING ONE PARAMETER AT A TIME

Parameter	Baseline	Low	High	Unit
sepMean	**80**	70	90	sec
sepStd	**18**	10	18	sec
yStd	**50**	25	50	ft
zStd	**50**	25	50	ft
vMean	**140**	130	140	kts
vStd	**12.5**	6	12.5	kts
e	**0.001**	0.001	0.01	m^2/s^3
N	**0**	0	0.04	1/s
windMean	**0/20***	0	10/20*	kts
windStd	**0**	0	5	kts
runwaySep*	**800**	800	1200	ft

* - used in parallel-runway scenario only

For space considerations, we present only a few representative results. Unless otherwise stated, all results are based on the TDAWP wake model and the weak wake threshold (75 m^2/s).

Figure 5 shows the effect of separation time on the simulated fraction of potential wake encounters. The sensitivity over the mean separation time is shown in the first figure. If the mean separation time is decreased (hypothetically) from 90 seconds to 70 seconds, the fraction of potential wake encounters increases, as expected. Holding the mean separation fixed, if the standard deviation is decreased from 18 seconds to 10 seconds (second figure), the fraction of potential wake encounters decreases. This is because a smaller standard deviation corresponds to a lower fraction of observations in which the separation time is small. In theory, it is possible to simultaneously reduce the separation mean and separation standard deviation (in some appropriate ratio) to keep the probability of a potential wake encounter constant.

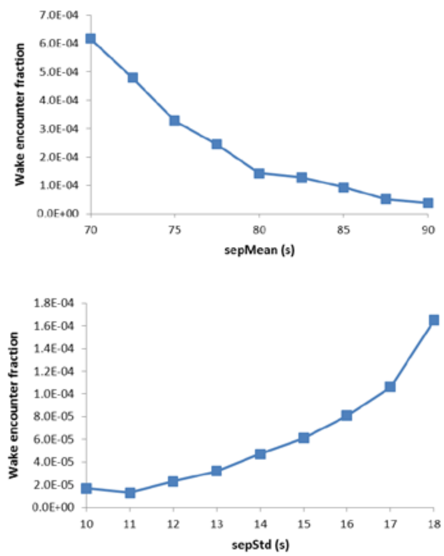

Figure 5. Sensitivity to separation-time average and standard deviation

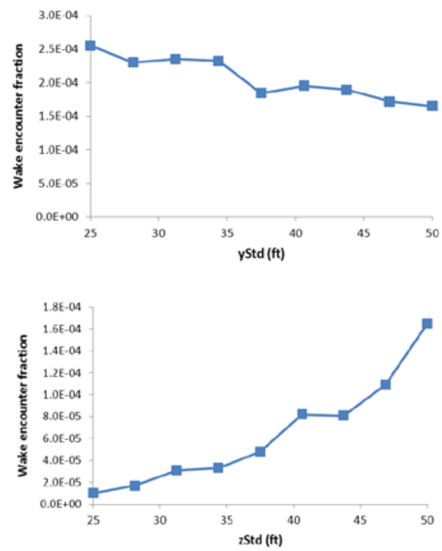

Figure 6. Sensitivity to lateral and vertical standard deviation

Figure 6 shows the effect of lateral and vertical standard deviation on the potential wake encounter fraction. The variability in the curves is due to the stochastic noise of the simulations. For a single-runway scenario, a larger lateral standard deviation leads to a lower probability of a potential wake encounter, because the leading and trailing aircraft are more likely to be located at different lateral positions. A tighter bound on the lateral position results in an increase in the probability, because the aircraft are more likely to be lined up close to the centerline. *These results assume a crosswind of 0 knots* (see Table II). A higher degree of vertical deviation leads to a higher probability of a potential wake encounter. This is because a higher vertical standard deviation leads to a higher probability that the trailing aircraft is low while the leading aircraft is high, resulting in the wake of the leading aircraft descending to the approximate height of the trailing aircraft.

Figure 7 shows the effect of crosswind on the single-runway airport model. Confidence intervals show the uncertainty due to finite simulation time (but do not show model uncertainty or parameter uncertainty). The top figure shows that higher mean wind speeds result in fewer potential wake encounters, since wake vortices are more likely to drift laterally from the centerline. The second figure shows that a higher standard deviation of crosswind (fixing the mean crosswind at zero) results in fewer potential wake encounters, since a higher variability results in a higher probability of a high crosswind. A limitation of varying one parameter at a time is that wind speed and eddy dissipation rate (EDR) tend to be correlated. Since we are assuming that EDR is fixed when varying the windspeed, these results may not be entirely representative of physical phenomena.

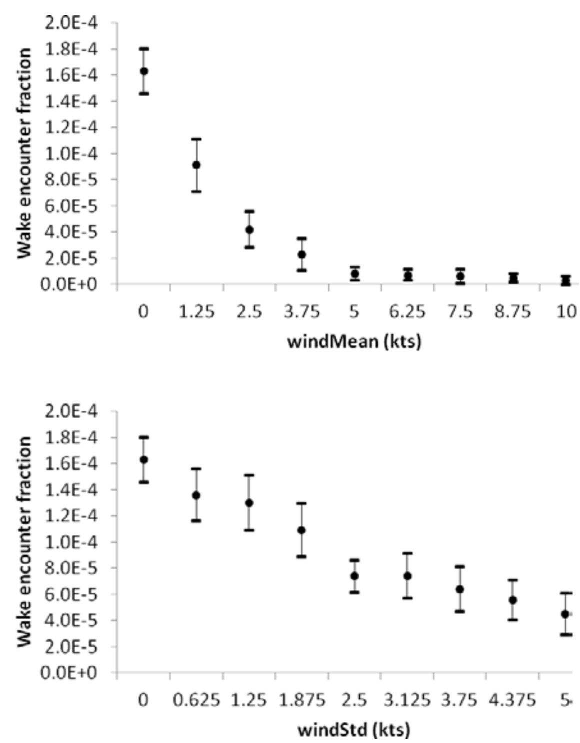

Figure 7. Crosswind effect on single runway wake encounter fractions

Figure 8 compares output between GMU implementations of the TDAWP and D2P models. The TDAWP model results in a relatively higher fraction of potential wake events than the D2P model. Nevertheless, they give similar trends in all cases. (Both experiments use the same sets of random variables for the flight tracks, which is why the bumps in the graphs occur at the same locations for both TDAWP and D2P. These bumps are due to stochastic variability in the simulation.)

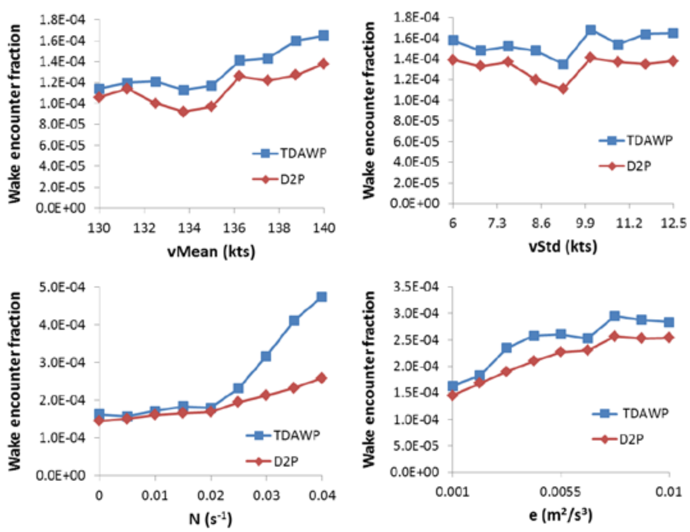

Figure 8. Comparison of outputs from TDAWP and D2P

Figure 9 shows interaction effects between the circulation threshold (the value that defines a potential wake encounter) and the EDR. For example, when considering high-circulation wakes (225 m^2/s or higher), a higher EDR results in fewer potential wake encounters. This is expected, since a higher EDR results in faster wake decay, resulting in fewer potential encounters. However, the trend is the opposite when low-circulation wakes (75 m^2/s or higher) are considered. A higher EDR results in faster wake decay, but this also corresponds to a slower sink rate. Thus, wakes are more likely to remain near the approach path, but in a weaker state. This is why the potential encounters increase with increasing EDR for a low circulation threshold of 75 m^2/s.

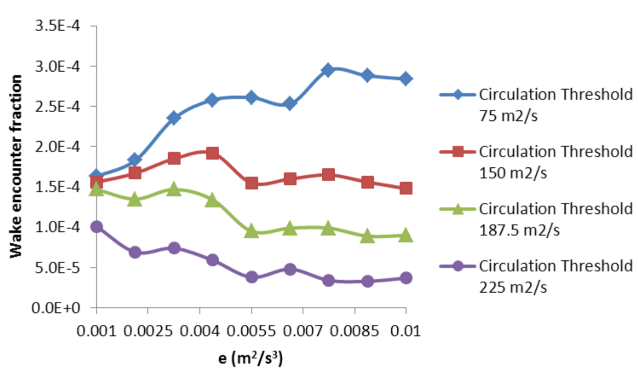

Figure 9. Interaction between EDR and specified circulation thresholds

B. Full Factorial Analysis

To study the interactions between parameters, a full factorial analysis is conducted. Having the quantitative impact of each parameter and the interaction between every pairwise combination of parameters provides a better picture of how variations in the input parameters combine to affect the overall probability of a potential wake encounter.

To simplify the problem, seven of the parameters are selected. Each parameter is set to either its low value or high value as given in Table III. All combinations of these values are studied. There are $2^7 = 128$ experiments conducted in total, each with 100,000 simulated flights. From the outputs of these experiments, we estimate the main effects (impacts of single factors) and interaction effects (impacts of pairs of factors). Because the input values are "normalized" to either a low value or a high value, the resulting effects provide some measure of the relative impact of the parameters and their interactions. Of course, the results are somewhat dependent on the choice of the low and high values.

TABLE III. DESIGN OF EXPERIMENTS, FULL FACTORIAL ANALYSIS

Parameter	Low	High	Unit
sepMean	70	90	sec
sepStd	10	18	sec
yStd	25	50	ft
zStd	25	50	ft
e	0.001	0.01	m^2/s^3
N	0	0.04	1/s
windMean	0	10/20*	kts
runwaySep*	800	1200	ft

* - used in parallel-runway scenario only

Figure 10 shows the main effects and interaction effects sorted by value. The corresponding parameter (or parameter pair) is labeled on the *y*-axis. The largest effects (positive or negative) are marked in ovals. These include the separation mean, separation standard deviation, vertical standard deviation, wind mean, and their interactions.

For example, to read this figure, the main effect for separation mean is negative (to the left). Thus, if one were to increase the separation mean, then the fraction of potential wake encounters would decrease as expected. In a similar manner, the main effect for the separation standard deviation is positive (to the right). Thus, if one were to increase the separation standard deviation, the potential wake encounters would increase. If one were to increase *both* the mean and the standard deviation of separation time, then the interaction effect must also be considered. In this case, the interaction effect is negative, which is in the same direction as the main effect for the separation mean. Roughly speaking, this means that if both the mean and standard deviation are increased, then the cumulative effect would be a decrease in potential wake encounters (a negative main effect from the separation mean, a positive effect from the separation standard deviation, and a negative joint effect). Said another way, if one were to *decrease* the separation mean and *decrease* the separation standard deviation, the beneficial effect of reducing the separation standard deviation would be somewhat negated by the negative effect of reducing the separation mean, resulting in an increase in potential wake encounters. It should be emphasized these results are all contigent upon the specific choices for the high and low parameter values and different choices might result in different parameter effects.

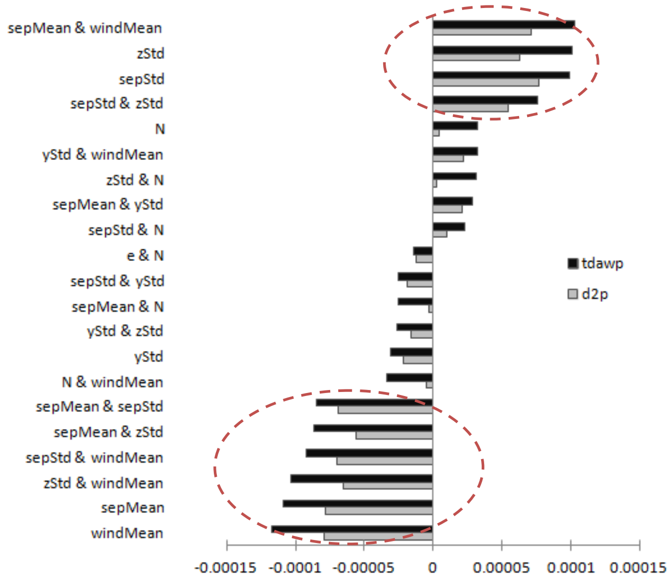

Figure 10. Full-factorial ranking of effects from TDAWP and D2P

The figure also compares results based on the TDAWP model and the D2P model. The numerical values of the coefficients are different – the D2P model tends to yield smaller parameter effects than the TDAWP model. Nevertheless, the ranking order for the impacts is nearly the same, especially for the dominant parameters.

IV. Parallel-Runway Model

This section considers a similar analysis for a parallel-runway scenario. In the analysis, the runways are spaced between 800 and 1,200 feet apart so that transport of a wake from one runway to the opposite runway can be considered.

To study the interactions between parameters, a full factorial analysis is conducted over eight parameters. Each parameter is set to either its low value or high value. All combinations of these values are studied. There are $2^8 = 256$ experiments conducted in total, each having 100,000 flights generated for simulation.

In the parallel-runway case, wake events can be attributed to either an aircraft on the same runway or an aircraft on the opposite runway. Since the same-runway case has effectively been considered in the previous section, this section only considers wake events attributed to the aircraft on the opposite runway. Aircraft on each runway are simulated in a similar manner as in the single-runway case. The two runways are simulated independently.

Figure 11 shows the impacts of each parameter and their pairwise interactions. This result is based on the TDAWP model using the weak wake threshold (75 m^2/s). The dominating factors are average crosswind speed, vertical standard deviation, runway separation, and their interactions. The average crosswind speed and vertical standard deviation and their interaction have a positive impact on wake encounters.

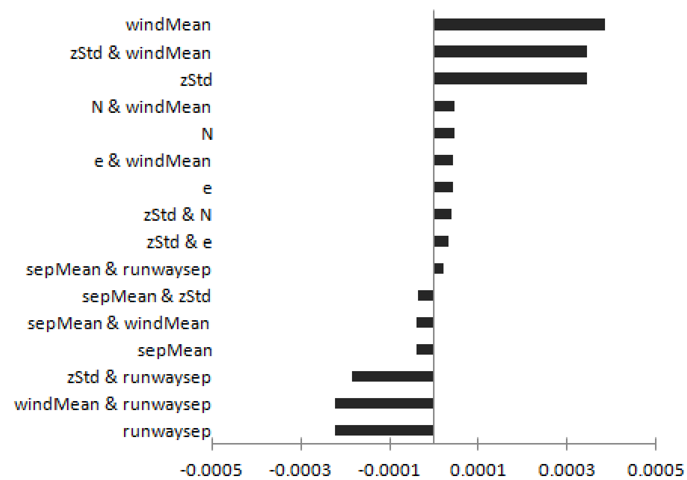

Figure 11. Dominating factors and their impacts on wake encounter probability

As in the single-runway case, the numerical values of the coefficients are different between the D2P and TDAWP models. Nevertheless, the ranking order for the impacts is nearly the same, especially for the dominating parameters.

V. Discussion

Some of the results given in the preceding sections are somewhat obvious and intuitive. For example, increasing the average along-track separation between aircraft reduces the probability of a potential wake encounter; increasing the spacing between runways also reduces the probability of a potential wake encounter. The contribution of this work is to (a) identify the parameters that have the greatest effect on potential wake-encounter probabilities and (b) understand the interaction effects.

For the single-runway model, key parameters are the average separation, the standard deviation of separation, and the vertical standard deviation. As expected, a proper trade-off must be maintained between the separation mean, which directly affects the associated capacity, and the standard deviation. The interaction effect shows that decreasing the separation mean may effectively cancel the benefit of decreasing the separation standard deviation.

From the models, decreasing the standard deviation of vertical position may have a greater impact on the probability of a potential wake encounter than decreasing the standard deviation of lateral position. This is because wakes tend to sink. Thus, it is easier to ensure separation between the trailing aircraft and the wake if the leading and trailing aircraft pass a given position at the same altitude. If there is high variability in the vertical dimension, it is more likely that the leading aircraft is high and the trailing aircraft is low.

Some non-obvious trade-offs are also revealed in the sensitivity analysis. In particular, it is possible to simultaneously increase the risk associated with high-strength vortices while lowering the risk associated with low-strength vortices, and vice versa (Figure 9). This illustrates the complexity of the physical dynamics and that a complete analysis must take into account the combined effects of potential wakes at a variety of circulation strengths.

Additional work is needed to evaluate the severity of wake encounter by considering the factors such as wake geometry characteristics and pilot reaction time. Also, more aircraft types should be included in the future, and correlations between parameters (e.g. crosswind vs. EDR) should be studied as well.

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