Why Equity is a Pipedream:
Analysis of the Dynamics of Overscheduled NAS Resources

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Abstract

One of the major issues in the proposed concepts-of-operations for NEXTGEN is the equitable allocation of overscheduled National Airspace System (NAS) resources. Although federal regulations, Congressional policies, and modernization plans call for equitable allocation of publicly held resources, the mechanisms for equitable allocation, and the trade-offs that must be made between stakeholders, exhibit a high degree of social, political, and economic complexity. Further, the allocations that are routinely made in Traffic Flow Management (TFM) and Air Traffic Control (ATC) are subject to widespread perceptions of systemic inequity and economic inefficiency.

This paper describes the properties associated with the dynamic allocation of an overscheduled resource. The mathematical model demonstrates that (i) a natural asymmetry exists in the allocation, (ii) this asymmetry results in a low probability (< 10%) of an equitable allocation of resources amongst groups of flights (e.g. airlines), and (iii) increased competition reduces the likelihood of equitable allocations. These theoretical results establish the "feasible space" in which claims of equity can be made, suggests refinements of the existing allocation schemes, and establishes the framework for a regulatory role in proposed market-based mechanisms.

Introduction

Agencies responsible for air transportation in Europe and the United States have announced long-term plans to significantly increase the capacity and productivity of the air transportation system through a process of modernization of the technologies and procedures used by Air Navigation Service Providers (ANSPs). In Europe the plans are known as SESAR [3]. In the United States the plans are known as NEXTGEN [1, 2].

These plans call for equitable access and use of public airspace and airport resources in several dimensions. For example, the plans for NextGen [1,2] describe "equitable treatment of flight operators" (pg. 2-16), "equitable access to NAS resources—both airports and airspace [for all users]" (pg 2-29), and "equitable management of system resources" (pg C-7).

Whereas these ideas are central to democratic ideals of governing in the "best interests of all citizens"[7], the reality of equitable allocation of scarce resources is elusive [6, 5] and involves trade-offs between stakeholders [8,9]. Although publicly available data is not available, the allocation of flight delays during Traffic Flow Management (TFM) initiatives (i.e. such as Ground Delay Programs and Airspace Flow Programs) is perceived to be economically inefficient and inequitable. As one senior ANSP manager put it, "we feel like we have done a good job when everybody is equally unhappy."

The objective of this paper is to determine the conditions that would have to exist to provide an equitable allocation of resources when a resource is overscheduled. Resources include arrival and departure slots, slots at metering fixes, as well as communication channels, gate usage, environmental cap-and-trade, etc. This paper focuses on the allocation of flights to slots. Equity refers to the proportional distribution of slots to groups of flights. Equity may be applied to flights grouped by airline to provide inter-airline equity. Equity may also be applied to flights grouped by: category of origin/destination (e.g. metropolitan, rural), by transportation productivity (e.g. number of seats per flight), by passenger/cargo value (e.g. business, leisure), by emissions or noise proclivity (e.g. clean engines), along with several other dimensions.

The main results of the analysis are as follows:

(1) <u>Asymmetry in Allocation</u> – the natural dynamics of the allocation of overscheduled resources results in the allocation of flights later in the overscheduled time period with higher delays than those in the front of the overscheduled period. Note: This property is the result of a "spill-over" that cascades through later flights.

(2) <u>Probability of Achieving Proportional Equity is Very Low</u> - equity for groups of flights, measured as the allocation of scarce resources in proportion to the number of flights in each group, can be achieved only by equal spacing of flights in the schedule such that each group is equally spread throughout the asymmetrical distribution. The probability of achieving equity through a randomly generated schedule is less than 10%.

(3) <u>Increasing Competition, Decreases Opportunities for Equity</u> – as the number of groups increases (i.e. increased competition), the probability of equity resulting from random scheduling decreases well below 10%.

These results establish the boundaries of a "feasible-space" in which equitable allocation can occur. The implications of these results are that equitable allocation of a randomly scheduled resource is close to impossible and claims of equity cannot be achieved on a systematic basis. Further the results suggest refinement of the allocation scheme (e.g. using exemptions and slot-swapping) to improve the equity. Lastly, the analysis motivates the migration to market-based mechanisms to manage a form of market-based equity.

This paper is organized as follows. The properties of the dynamics of the allocation overscheduled resources are described in Section 2. Section 3 provides a definition of proportional equity and describes the properties of equitable allocation. Section 4 uses these properties to derive the probability of an equitable allocation amongst groups of randomly generated schedules in excess of the capacity of the resource. The implications of these results are discussed in the Conclusions, Section 5.

Dynamical Properties of the Allocation of Overscheduled Resources

A canonical representation of an over-scheduled resource is illustrated in Figure 1. The resource is flight slots (e.g. arrival slots) that are grouped in 15 minutes periods. Starting at 15 minute period #6, flights are scheduled in excess of the *available capacity* for the duration of a period labeled as the "congested period." Following the congested period, the *low-demand* provides a reservoir to absorb spill-over delays. The flights are numbered in the order in which they are scheduled.

The systemic allocation of delays to individual flights according to the order of scheduling is illustrated in Figure 2. The color-code identifies the magnitude of delay experienced by each flight. Flights in excess of the capacity in the first 15 minute period, spill-over into the second 15 minute period escalating the delays assigned to these flights. The spill-over cascades through the overscheduled period (15 minute time slots 6 through 15) and into the period following the overscheduled period (15 minute time slots 16 through 25).
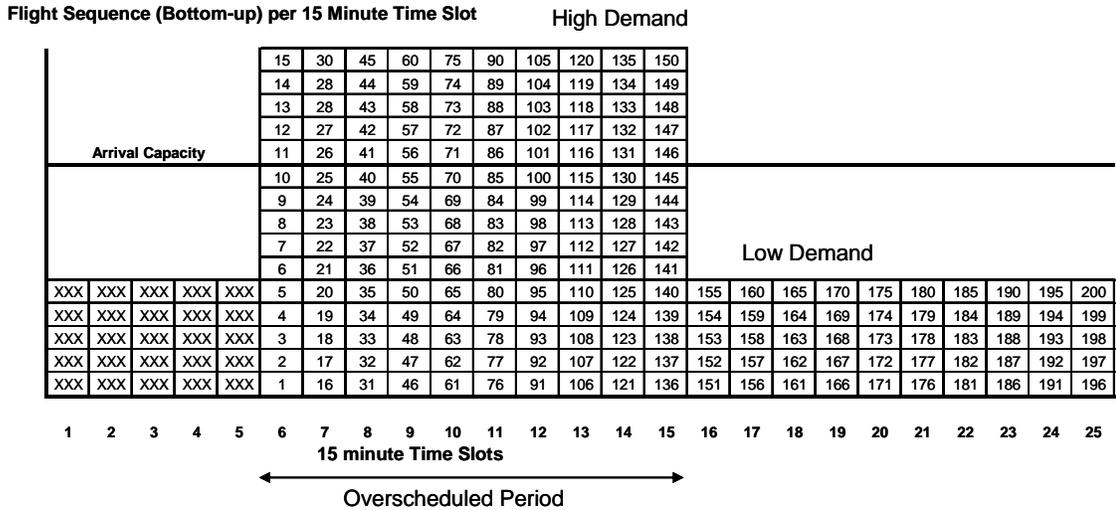
Dynamical Properties

The sum of the individual flight delays, known as the Total Delay Time, is a function of the magnitude of over-scheduling as well as the capacity of the reservoir following the over-scheduled period. As shown in Figure 3, a queue of flights builds until the end of the overscheduled period, at which time the reservoir of capacity allows the queue to dissipate.

The Total Delay Time resulting from the scheduled utilization is defined by the equation:

Total Delay Time =

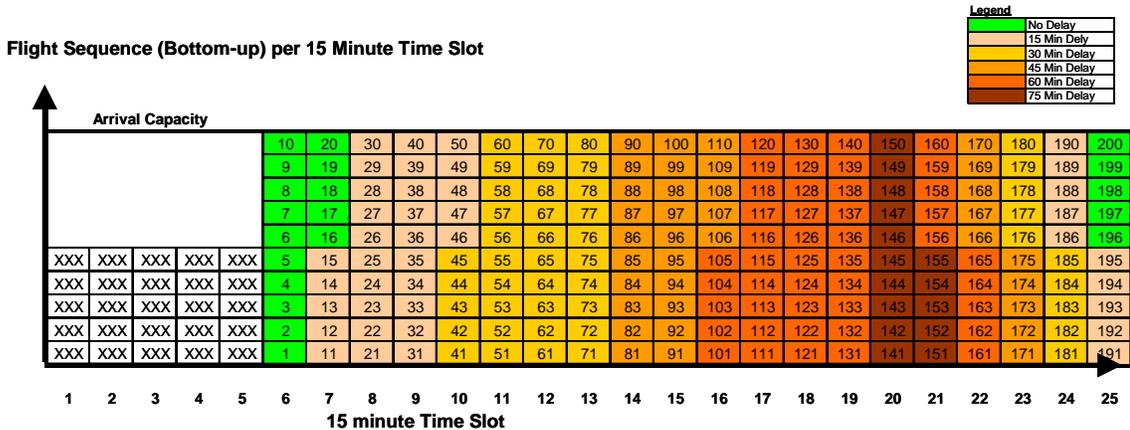½ * (Duration of Overscheduled Period)^2*

(HighDemand – Capacity) *

[1 + $\frac{\text{HighDemand} - \text{Capacity}}{\text{Capacity} - \text{LowDemand}}$]

The equation above highlights the following properties of the dynamics of overscheduled

Flight Sequence (Bottom-up) per 15 Minute Time Slot

					15	30	45	60	75	90	105	120	135	150											
					14	28	44	59	74	89	104	119	134	149											
					13	28	43	58	73	88	103	118	133	148											
					12	27	42	57	72	87	102	117	132	147											
Arrival Capacity					11	26	41	56	71	86	101	116	131	146											
					10	25	40	55	70	85	100	115	130	145											
					9	24	39	54	69	84	99	114	129	144											
					8	23	38	53	68	83	98	113	128	143											
					7	22	37	52	67	82	97	112	127	142											
					6	21	36	51	66	81	96	111	126	141											
XXX	XXX	XXX	XXX	XXX	5	20	35	50	65	80	95	110	125	140	155	160	165	170	175	180	185	190	195	200
XXX	XXX	XXX	XXX	XXX	4	19	34	49	64	79	94	109	124	139	154	159	164	169	174	179	184	189	194	199
XXX	XXX	XXX	XXX	XXX	3	18	33	48	63	78	93	108	123	138	153	158	163	168	173	178	183	188	193	198
XXX	XXX	XXX	XXX	XXX	2	17	32	47	62	77	92	107	122	137	152	157	162	167	172	177	182	187	192	197
XXX	XXX	XXX	XXX	XXX	1	16	31	46	61	76	91	106	121	136	151	156	161	166	171	176	181	186	191	196

Low Demand

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
15 minute Time Slots

Overscheduled Period

Over-schedule of a resources (e.g. arrival slots).
Figure 1

Flight Sequence (Bottom-up) per 15 Minute Time Slot

Legend
No Delay
15 Min Dely
30 Min Delay
45 Min Delay
60 Min Delay
75 Min Delay

Arrival Capacity

					10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
					9	19	29	39	49	59	69	79	89	99	109	119	129	139	149	159	169	179	189	199
					8	18	28	38	48	58	68	78	88	98	108	118	128	138	148	158	168	178	188	198
					7	17	27	37	47	57	67	77	87	97	107	117	127	137	147	157	167	177	187	197
					6	16	26	36	46	56	66	76	86	96	106	116	126	136	146	156	166	176	186	196
XXX	XXX	XXX	XXX	XXX	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195
XXX	XXX	XXX	XXX	XXX	4	14	24	34	44	54	64	74	84	94	104	114	124	134	144	154	164	174	184	194
XXX	XXX	XXX	XXX	XXX	3	13	23	33	43	53	63	73	83	93	103	113	123	133	143	153	163	173	183	193
XXX	XXX	XXX	XXX	XXX	2	12	22	32	42	52	62	72	82	92	102	112	122	132	142	152	162	172	182	192
XXX	XXX	XXX	XXX	XXX	1	11	21	31	41	51	61	71	81	91	101	111	121	131	141	151	161	171	181	191

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
15 minute Time Slot

Flights allocated slots within the available capacity. Note that the effect of over-scheduling causes spill-over resulting in increasing delays to flights later in the schedule. Delays also spill-over into the period after the over-scheduled period.
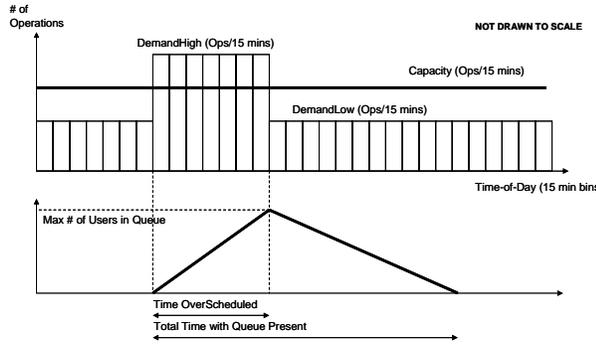*Figure 2.*

resources that are *independent* of the method of allocation:

P1 Conservation of Total Delays. The Total Delay (i.e. sum of delay to each flight) is independent of the order of the allocation of the individual flights. Alternate allocations result in the same Total Delay.

P2 Magnitude of Total Delays The Total Delay is dependent on the relationship amongst the four terms: Capacity, High_Demand, Low_Demand and Overscheduled_Period.

P3 Duration of Overscheduled Period The factor in the equation that has the biggest effect is the duration of the Overscheduled Period. This term is squared. For every additional period in the Overscheduled period, the Total Delays increase geometrically.

P4 Reservoirs are Critical: The Total Delay is not only dependent on the degree of over-scheduling, but also on the degree of under-scheduling after the overscheduled period. The degree of under-scheduling provides a reservoir to absorb the spill-over from the overscheduled

3

Queueing of flights due to over-scheduling. The Total Delay time is the area under the Queueing function.

Figure 3

period. A low degree of under-scheduling can result in extending the queue significantly.

The delays experienced by individual flights for the example in Figure 1 and 2 are shown in Figure 4. The x-axis is the ordered list of flights. The y-axis is the delay assigned to each flight by a First-Scheduled/First-Allocated procedure. Flights early in the overscheduled period experience relatively low delays. Flights at the end of the overscheduled period and flights right after the overscheduled period experience the highest delays. This phenomenon defines an additional property of overscheduled resources:

P5   Asymmetry of Individual Flight Delays: The delays assigned to individual flights are a function of the location of the flight in the schedule. Flights scheduled early in the overscheduled period, are allocated less delays than those flights later in the overscheduled period.
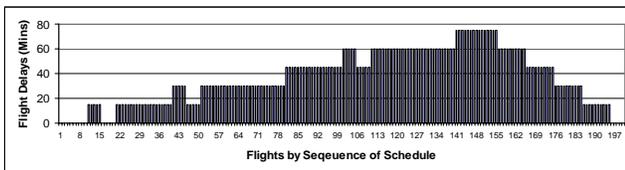
Delays experienced by individual flights are asymmetrical. Flights deeper in the overscheduled period (and right after the overscheduled period) are assigned more delay than those at the front.

Figure 4.

Equity in Allocation of Overscheduled Resources

Every society has rules for allocating resources and burdens among it's members [12]. There are two general classes of allocation rules:

(1) resources are allocated through property rights and liabilities that are held and traded by private individuals or held by enterprises according to complex financial regulations. Market-based mechanisms ensure an economically efficient allocation and rapid adjustment to changes in the environment, but are subject to inequitable social distributions

(2) property rights for the resources are held by a governing entity and allocated according to societal needs. Regulated distribution can overcome the societal distortions of market-based mechanisms but are politically driven and slow to adapt.

The rules for allocation of centrally held property rights requires three decisions: (1) the *supply decision* determines the amount of resources to be distributed (e.g. arrival slots). (2) the *distributive decision* determines the principles and methods used to allocate the resources (e.g. first-scheduled/first-served), and (3) the *reactive decision*: determines the owners or users to the allocation scheme (e.g. slot substitutions and slot swapping). The focus of this paper is the implications of the distributive decision.

The mechanism for the distribution of property rights expresses the notions of equity in the division of the resources deemed reasonable by societal norms. The appropriateness of the equity is determined in part by principle and in part by precedent.

Equitable allocation is conducted through a process in which each party is allocated an equal distribution measured according to a single yardstick. There are two issues: First, the reality is that allocated resources are not equal. Second, the claimant parties are in different situations, and that the agreement of a single yardstick is difficult to achieve [4].

A wide range of philosophers (e.g. Aristotle, Maimonides) have examined the "combinatorics" of allocation of asymmetric resources to claimants using various yardsticks and have developed

appropriate allocation schemes for specific combinations. One of the emergent themes of these allocation schemes is that the equity formulas are usually based, either explicitly or implicitly, on a standard of comparison that ranks the various claimants on their relative entitlement [12; pg 80].

Proportionality and Proportional Equity

One of the oldest and most widely used distributive principles is one that ranks claimants rights. This is the "Principle of Proportionality." Proportionality is implicit in the mechanism of First-Come/First-Serve used in Air Traffic Control (ATC) and is the explicit in the mechanism of First-Scheduled/First-Served used in Traffic Flow Management (TFM). This principle allocates the resources in proportion to the demand for the resources such that groups of flights (e.g. airlines) will receive delays in proportion to the number of flights they have scheduled.

Proportional Equity is defined by the following equation:

Proportional Equity for Group (i) =

$$\frac{[\text{Total Delay Group (i) / Total Delay }]}{[\text{\# Flights Group (i) / Total Flights }]}$$

where i are groups of users (e.g. airlines) 1 through N.

The numerator represents the proportion of delays allocated to Group (i) with respect to the total delays allocated to all the Groups. The denominator represents the proportion of flights flown by Group (i).

When the proportion of delays is equivalent to the proportion of flights, the equity for the group is equal to 1. Proportional equity less than 1, implies that the group was allocated delays proportionately less than the number of flights scheduled. This group benefited from the allocation process. Proportional equity greater that 1, implies that the group was allocated delays proportionately more than the number of flights scheduled. This group was penalized by the allocation process. Note: an alternative representation of proportional equity was proposed by Manley & Sherry [9] that is based on the log 10 and provides non-linear scale referenced to zero.

Proportional Equity for Overscheduled Resources

Proportional Equity for N groups of flights is achieved when the ratio of % delays for each group and the % flights for each group, approaches unity. Equity amongst groups can only be achieved when the flights in each group are symmetrically located around the centroid for the individual flight delays. When the N groups have equal number of flights, the centroid is the average flight delay. When the N groups have different number of flights, the centroid is the average delay allocated to each group for all possible allocations for the group. The expected value is known as the Shapely value.

Due to the asymmetry of delays assigned to individual flights (described above), proportional equity for groups of flights is achieved only when the flights are allocated evenly around the slots that are allocated the average delay.

Take for example the case of 8 flights that are scheduled two at a time into 4 flight slots. An allocation by first-scheduled/first-allocated scheme would spread the flights into 8 slots in the order they were scheduled. The Total Delay is 16 slots. The individual flight delay ranges from 0 to 4 slots. For two groups of flights, the equitable allocation would be 4 delays slots per group (i.e. 16/(2*2 groups) = 4). Proportional equity is achieved when each group has two flights on either side of the centroid with symmetric slots (e.g. first and last, second and second-last).

P6   Equal and Rigid Spacing amongst Groups Ensures Equity: Equity between groups is achieved when the individual flights in each group are allocated with equal slot spacing throughout the allocation.

Probability of Equitable Allocation for Randomly Scheduled Flights

Flights are scheduled by the individual airlines to meet the airlines network, connecting passenger, and connecting crew objectives, and are constrained by the available airline resources. When flights are scheduled by independent airlines, without consideration of the capacity of the resource and without *a priori* knowledge of the schedules of other airlines, over-scheduling can occur. Due to
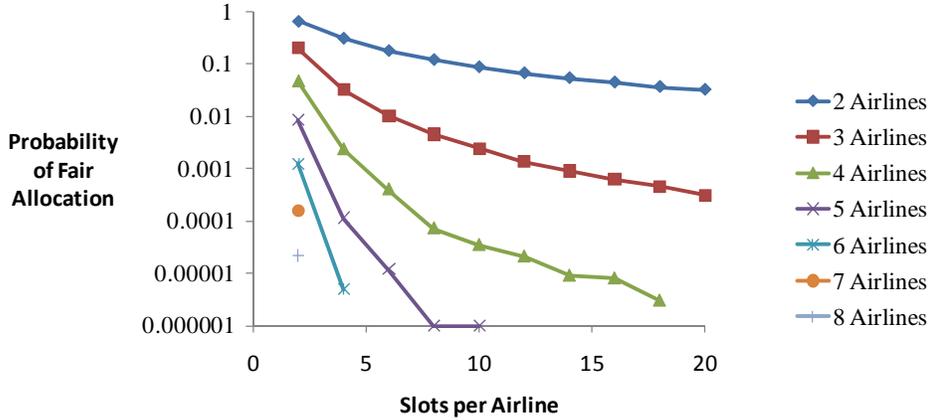
Results of simulation experiments to estimate the probabilities of proportional allocation of slots. Each data point is estimated using one million random slot allocations. The x-axis represents the number of slots per airline and the y-axis shows the probability of a proportionally equitable allocation on a logarithmic scale.

Figure 6

airline scheduling constraints it is unlikely that airlines schedules would result in symmetric slot allocation that would result in an equitable allocation.

The question posed by this model of over-scheduled resources is *what is the probability that a cumulative schedule (without any structure imposed) will yield proportional equity for any number of groups*?

One way to determine the probability that a random allocation of slots is proportionally equitable is to enumerate all possible slot allocations and then to count the fraction of such allocations that are equitable. The combinatorial explosion in the number of possible schedules for more than a dozen flights makes this approach infeasible. An alternative approach is to estimate the probability of equitable allocations through Monte-Carlo computer simulation. An algorithm was developed for random generation of schedules and the subsequent determination of whether or not the slot assignment is equitable. Uniform distributions are used to ensure breadth of coverage. By repeating the algorithm multiple times and counting the fraction of times that equitable allocations are generated, the probability of an equitable allocation can be estimated [13] for information on the algorithm.

Figure 6 shows the results of simulation experiments to estimate the probabilities of proportional allocation of delays to randomly generated schedules. Each data point represents a combination of number of airlines (N) and number of flights per airline (M). The total number of flights in each schedule is N x M. One million randomly generated scheduled were evaluated for each data point.

The *x*-axis represents the number of slots per airlines. The color-coded data-points represent the number of airlines. The *y*-axis shows the probability of a proportionally equitable allocation for one million randomly generated schedules. The y-axis uses a logarithmic scale. The threshold for proportional equity was between 0.9 and 1.1. (i.e. +/-10% range).

These results indicate the elusiveness of individual groups (e.g. airlines, passenger demographics, aircraft/engine type) of randomly scheduling such that proportional equity can be achieved. The probability of obtaining an equitable allocation decreases both in the number of slots per airline and the number of airlines.

The probability decreases extremely fast in the number of airlines. Since *every* airline must have the same total delay, there are more constraints on the situation when there are more airlines. For example, if 3 airlines are involved and each airline

has 4 slots, the probability of proportional equity is less than 1%. If 4 airlines are involved, the value drops to about .01%. If 5 or more airlines are involved, the probability is less than one in a million (no schedules with proportional equity were observed in the simulations). In fact, the only cases in which a "reasonable" probability of a proportional equity is observed are cases involving 2 airlines, and this is still typically below 10% and closer to 1%.

Conclusions and Future Work

When a centralized entity is responsible for allocation of overscheduled resources, the degrees of authority to allocate the flights with proportional equity are limited by the symmetry of schedules of the groups. The low probability of achieving equity suggests alternate approaches:

1) Market-based mechanisms: Slot assignment using this approach ensures an economically efficient use of the resource. Criticism of this approach is that societal needs can be ignored and require regulatory intervention. One example is Essential Air Services program, that subsidizes air service to rural markets that cannot sustain service by profit-seeking airlines that operate in a de-regulated marketplace. See [10, 11].

2) Refining First-Scheduled/First-Allocated to Improve Equity: TFM initiatives are designed to "exempt" flights from delay allocations (i.e. these flights maintain their original scheduled slot) and "compress," "swap" and "substitute" flights to improve their position in the allocation. These adjustments provide mechanisms to adjust the allocation thereby adjusting the equity. Future work is required to establish the degree to which these adjustments can be made

3) Temporal Measures of Equity: Instead of measuring equity over a single allocation, equity could be measured across multiple allocations (e.g. days). Given the degree of accuracy in which flights can be controlled to arrive at the constrained resource, the equity may even out over time. The degree to which normal arrival errors adjust affect equity is a subject for future work.

4) Variable Allocation Schemes: Combined with temporal measures of equity, variable allocation schemes may result in more equitable allocations.

5) Hybrid Allocation Schemes: A market-based allocation scheme with regulatory constraints may yield the most economically efficient and socially equitable allocation.

Future Work
Along with the issues listed above, the following topics are considered for future work:

1) Complete the closed form solution or hybrid to enumerate the full combinatorics of the grouping of flights.

2) Use exemptions and slot swapping to balance out large inequities.

3) Use asymmetrical compression to balance out equity.

4) Equity measured on multiple dimensions. See Schaar & Sherry [8].

5) Multi-dimensional equity required to truly trade-off best interests to society.

6) Underlying "structure" of fleet mix and route structures bound the state-space in which the equity trade-offs can be made.

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