

Why Equity is Elusive: Dynamical Properties of Overscheduled National Airspace System Resources

Lance Sherry¹, John Shortle², Vivek Kumar³,

Center for Air Transportation Systems Research, George Mason University, Fairfax, VA, 22030

One of the major issues in the design of the concepts-of-operations for NEXTGEN is the equitable allocation of overscheduled National Airspace System (NAS) resources. Although federal regulations, Congressional policies, and modernization plans call for equitable allocation of publicly held resources, the mechanisms for equitable allocation, and the trade-offs that must be made between stakeholders, exhibit a high degree of social, political, and economic complexity. Further, the allocations that are routinely made in Traffic Flow Management (TFM) and Air Traffic Control (ATC) are subject to widespread perceptions of systemic inequity and economic inefficiency. This paper describes the properties associated with the dynamic allocation of an overscheduled resource. The mathematical model demonstrates that (i) a natural equity asymmetry exists in the first-scheduled/first-allocated mechanism, (ii) this asymmetry results in a low probability (< 10%) of an equitable allocation of resources amongst groups of flights (e.g. airlines), and (iii) increased competition reduces the likelihood of equitable allocations. These theoretical results establish the "feasible space" in which claims of equity can be made, suggests refinements of the existing allocation schemes, and establishes the framework for a regulatory role in proposed market-based mechanisms.

I. Introduction

Agencies responsible for air transportation in Europe and the United States have announced long-term plans to significantly increase the capacity and productivity of the air transportation system through a process of modernization of the technologies and procedures used by Air Navigation Service Providers (ANSPs). In Europe the plans are known as SESAR (Ref 3) In the United States the plans are known as NEXTGEN (Ref 12, 2).

These plans call for equitable access and use of public airspace and airport resources in several dimensions. For example, the plans for NextGen (Ref 1,2) describe "equitable treatment of flight operators" (pg. 2-16), "equitable access to NAS resources—both airports and airspace [for all users]" (pg 2-29), and "equitable management of system resources" (pg C-7).

Whereas these ideas are central to democratic ideals of governing in the "best interests of all citizens" (Ref 7), the reality of equitable allocation of scarce resources is elusive (Ref 5, 6) and involves trade-offs between stakeholders (Ref 8, 9). Although publicly available data is not available, the allocation of flight delays during Traffic Flow Management (TFM) initiatives (i.e. such as Ground Delay Programs and Airspace Flow Programs) is perceived to be economically inefficient and inequitable. As one senior ANSP manager put it, "we feel like we have done a good job when everybody is equally unhappy."

The objective of this paper is to determine the conditions that would have to exist to provide an equitable allocation of resources when a resource is overscheduled. Resources include arrival and departure slots, slots at metering fixes, as well as communication channels, gate usage, environmental cap-and-trade, etc. This paper focuses on the

¹ Assoc. Professor, System Engineering and Operations Research, 4400 University Drive, MS 4A6, Fairfax, VA, 22030 AIAA Member.

² Assoc. Professor, System Engineering and Operations Research, 4400 University Drive, MS 4A6, Fairfax, VA, 22030.

³ Research Engineer, Center for Air Transportation Systems Research, 4400 University Drive, MS 4A6, Fairfax, VA, 22030.

allocation of flights to slots. Equity refers to the proportional distribution of slots to groups of flights. Equity may be applied to flights grouped by airline to provide inter-airline equity. Equity may also be applied to flights grouped by: category of origin/destination (e.g. metropolitan, rural), by transportation productivity (e.g. number of seats per flight), by passenger/cargo value (e.g. business, leisure), by emissions or noise proclivity (e.g. clean engines), along with several other dimensions.

The main results of the analysis are as follows:

- (1) Asymmetry in Allocation – the natural dynamics of the allocation of overscheduled resources using the first-scheduled/first-allocated mechanism results in increasing delays allocated to flights later in the overscheduled time period than those flights in the front of the overscheduled period. Note: This property is the result of a “spill-over” that cascades through later flights.
- (2) Probability of Achieving Proportional Equity is Very Low - equity for groups of flights, measured as the allocation of scarce resources in proportion to the number of flights in each group, can be achieved only by equal spacing of flights in the schedule such that each group is equally spread throughout the asymmetrical distribution. The probability of achieving equity through a randomly generated schedule is less than 10%.
- (3) Increasing Competition, Decreases Opportunities for Equity – as the number of groups increases (i.e. increased competition), the probability of equity resulting from random scheduling decreases well below 10%.

These results establish the boundaries of a “feasible-space” in which equitable allocation can occur. The implications of these results are that equitable allocation of a randomly scheduled resource is close to impossible and claims of equity cannot be achieved on a systematic basis. Further the results suggest refinement of the allocation scheme (e.g. using exemptions and slot-swapping) to improve the equity. Lastly, the analysis motivates the migration to market-based mechanisms to manage a form of market-based equity.

This paper is organized as follows. The properties of the dynamics of the allocation overscheduled resources are described in Section 2. Section 3 provides a definition of proportional equity and describes the properties of equitable allocation. Section 4 uses these properties to derive the probability of an equitable allocation amongst groups of randomly generated schedules in excess of the capacity of the resource. The implications of these results are discussed in the Conclusions, Section 5.

II. Dynamical Properties of the Allocation of Overscheduled Resources

A canonical representation of an over-scheduled resource is illustrated in Figure 1. The resource is flight slots (e.g. arrival slots) that are grouped in 15 minutes periods. Starting at 15 minute period #6, flights are scheduled in excess of the *available capacity* for the duration of a period labeled as the “congested period.” Following the congested period, the *low-demand* provides a reservoir to absorb spill-over delays. The flights are numbered in the order in which they are scheduled.

The systemic allocation of delays to individual flights according to the order of scheduling is illustrated in Figure 2. The color-code identifies the magnitude of delay experienced by each flight. Flights in excess of the capacity in the first 15 minute period, spill-over into the second 15 minute period escalating the delays assigned to these flights. The spill-over cascades through the overscheduled period (15 minute time slots 6 through 15) and into the period following the overscheduled period (15 minute time slots 16 through 25).

A. Dynamical Properties

The sum of the individual flight delays, known as the Total Delay Time, is a function of the magnitude of over-scheduling as well as the capacity of the reservoir following the over-scheduled period. As shown in Figure 3, a queue of flights builds until the end of the overscheduled period, at which time the reservoir of capacity allows the queue to dissipate.

The Total Delay Time resulting from the scheduled utilization is defined by the equation:

Property #1 Conservation of Total Delays. The Total Delay (i.e. sum of delay to each flight) is independent of the order of the allocation of the individual flights. Alternate allocations result in the same Total Delay.

Property #2 Magnitude of Total Delays The Total Delay is dependent on the relationship amongst the four terms: Capacity, High_Demand, Low_Demand and Overscheduled_Period.

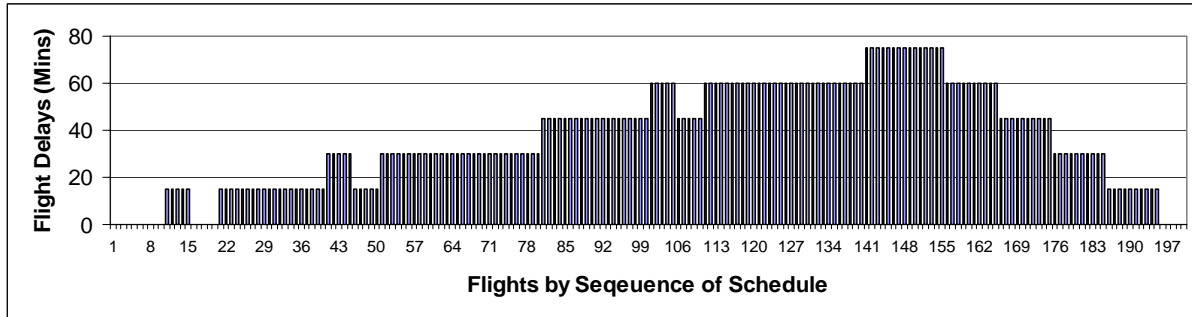


Figure 4: Delays experienced by individual flights are asymmetrical. Flights deeper in the overscheduled period (and right after the overscheduled period) are assigned more delay than those at the front.

Property #3 Duration of Overscheduled Period The factor in the equation that has the biggest effect is the duration of the Overscheduled Period. This term is squared. For every additional period in the Overscheduled period, the Total Delays increase geometrically.

Property #4 Reservoirs are Critical: The Total Delay is not only dependent on the degree of over-scheduling, but also on the degree of under-scheduling after the overscheduled period. The degree of under-scheduling provides a reservoir to absorb the spill-over from the overscheduled period. A low degree of under-scheduling can result in extending the queue significantly.

The delays experienced by individual flights for the example in Figure 1 and 2 are shown in Figure 4. The x-axis is the ordered list of flights. The y-axis is the delay assigned to each flight by a First-Scheduled/First-Allocated procedure. Flights early in the overscheduled period experience relatively low delays. Flights at the end of the overscheduled period and flights right after the overscheduled period experience the highest delays. This phenomenon defines an additional property of overscheduled resources:

Property #5 Asymmetry of Individual Flight Delays: The delays assigned to individual flights are a function of the location of the flight in the schedule. Flights scheduled early in the overscheduled period, are allocated less delays than those flights later in the overscheduled period.

III. Equity in Allocation of Overscheduled Resources

Every society has rules for allocating resources and burdens among it's members (Ref 12). There are two general classes of allocation rules:

- (1) resources are allocated through property rights and liabilities that are held and traded by private individuals or held by enterprises according to complex financial regulations. Market-based mechanisms ensure an economically efficient allocation and rapid adjustment to changes in the environment, but are subject to inequitable social distributions
- (2) property rights for the resources are held by a governing entity and allocated according to societal needs. Regulated distribution can overcome the societal distortions of market-based mechanisms but are politically driven and slow to adapt.

The rules for allocation of centrally held property rights requires three decisions: (1) the *supply decision* determines the amount of resources to be distributed (e.g. arrival slots). (2) the *distributive decision* determines the principles

and methods used to allocate the resources (e.g. first-scheduled/first-served), and (3) the *reactive decision*: determines the owners or users to the allocation scheme (e.g. slot substitutions and slot swapping). The focus of this paper is the implications of the distributive decision.

The mechanism for the distribution of property rights expresses the notions of equity in the division of the resources deemed reasonable by societal norms. The appropriateness of the equity is determined in part by principle and in part by precedent.

Equitable allocation is conducted through a process in which each party is allocated an equal distribution measured according to a single yardstick. There are two issues: First, the reality is that allocated resources are not equal. Second, the claimant parties are in different situations, and that the agreement of a single yardstick is difficult to achieve (Ref 4).

A wide range of philosophers (e.g. Aristotle, Maimonides, Shapely) have examined the “combinatorics” of allocation of asymmetric resources to claimants using various yardsticks and have developed appropriate allocation schemes for specific combinations. One of the emergent themes of these allocation schemes is that the equity formulas are usually based, either explicitly or implicitly, on a standard of comparison that ranks the various claimants on their relative entitlement (Ref 12; pg 80).

B. Proportionality and Proportional Equity

One of the oldest and most widely used distributive principles is one that ranks claimants rights. This is the “Principle of Proportionality.” Proportionality is implicit in the mechanism of First-Come/First-Serve used in Air Traffic Control (ATC) and is the explicit in the mechanism of First-Scheduled/First-Served used in Traffic Flow Management (TFM). This principle allocates the resources in proportion to the demand for the resources such that groups of flights (e.g. airlines) will receive delays in proportion to the number of flights they have scheduled. Proportional Equity is defined by the following equation:

$$\text{Proportional Equity for Group (i)} = \frac{[\text{Total Delay Group (i)} / \text{Total Delay}]}{[\text{\# Flights Group (i)} / \text{Total Flights}]}$$

where i are groups of users (e.g. airlines) 1 through N.

The numerator represents the proportion of delays allocated to Group (i) with respect to the total delays allocated to all the Groups. The denominator represents the proportion of flights flown by Group (i).

When the proportion of delays is equivalent to the proportion of flights, the equity for the group is equal to 1. Proportional equity less than 1, implies that the group was allocated delays proportionately less than the number of flights scheduled. This group benefited from the allocation process. Proportional equity greater than 1, implies that the group was allocated delays proportionately more than the number of flights scheduled. This group was penalized by the allocation process. Note: an alternative representation of proportional equity was proposed by Manley & Sherry (Ref 9) that is based on the log 10 and provides non-linear scale referenced to zero.

C. Proportional Equity for Overscheduled Resources

Proportional Equity for N groups of flights is achieved when the ratio of % delays for each group and the % flights for each group, approaches unity. Equity amongst groups can only be achieved when the flights in each group are symmetrically located around the centroid for the individual flight delays. When the N groups have equal number of flights, the centroid is the average flight delay. When the N groups have different number of flights, the centroid is the average delay allocated to each group for all possible allocations for the group. The expected value is known as the Shapely value.

Due to the asymmetry of delays assigned to individual flights (described above), proportional equity for groups of flights is achieved only when the flights are allocated evenly around the slots that are allocated the average delay.

Take for example the case of 8 flights that are scheduled two at a time into 4 flight slots. An allocation by first-scheduled/first-allocated scheme would spread the flights into 8 slots in the order they were scheduled. The Total Delay is 16 slots. The individual flight delay ranges from 0 to 4 slots. For two groups of flights, the equitable allocation would be 4 delays slots per group (i.e. $16/(2*2 \text{ groups}) = 4$). Proportional equity is achieved when each group has two flights on either side of the centroid with symmetric slots (e.g. first and last, second and second-last). Property #6 Equal and Rigid Spacing amongst Groups Ensures Equity: Equity between groups is achieved when the individual flights in each group are allocated with equal slot spacing throughout the allocation.

IV. Probability of Equitable Allocation for Randomly Scheduled Flights

Flights are scheduled by the individual airlines to meet the airlines network, connecting passenger, and connecting crew objectives, and are constrained by the available airline resources. When flights are scheduled by independent airlines, without consideration of the capacity of the resource and without *a priori* knowledge of the schedules of other airlines, over-scheduling can occur. Due to airline scheduling constraints it is unlikely that airlines schedules would result in symmetric slot allocation that would result in an equitable allocation.

The question posed by this model of over-scheduled resources is *what is the probability that a cumulative schedule (without any structure imposed) will yield proportional equity for any number of groups?*

One way to determine the probability that a random allocation of slots is proportionally equitable is to enumerate all the possible slot allocations and then to count the fraction of such allocations that are equitable. The combinatorial explosion in the number of possible schedules for more than a ten flights makes this approach infeasible. An alternative approach is to estimate the probability of equitable allocations through Monte-Carlo computer simulation. An algorithm was developed for random generation of schedules and the subsequent determination of whether or not the slot assignment is equitable. Uniform distributions are used to ensure breadth of coverage. By repeating the algorithm multiple times and counting the fraction of times that equitable allocations are generated, the probability of an equitable allocation can be estimated. See (Ref 13) for information on the algorithm.

Figure 6 shows the results of simulation experiments to estimate the probabilities of proportional allocation of delays to randomly generated schedules. Each data point represents a combination of the number of airlines (N) and the total number of flights in the schedule (N * M flights per airline). The results shown here are for one million randomly generated schedules. The threshold for proportional equity was between 0.9 and 1.1. (i.e. +/-10% range).

The y-axis represents the probability of an equitable allocation. The x-axis defines the total number of flights scheduled. Each curve represents the number of airlines in each simulation. These results indicate the elusiveness of individual groups (e.g. airlines, passenger demographics, aircraft/engine type) of randomly scheduling such that proportional equity can be achieved. The probability of obtaining an equitable allocation decreases both in the number of slots per airline and the number of airlines.

Property # 6. Increasing number of flights, decreases the probability of an equitable solution

As the number of flights increases, the number of permutations of allocations per airline increases. The increase in number of permutations of allocations per airline causes the probability of an equitable solution to decrease. Take for example, the two-airline/four-flight scenario, the total delays experienced is 4 slots. There are 6 ways the airlines flights can be grouped, with four yielding an equitable allocation. For the two-airline/eight-flight scenario, the probability of an equitable allocation drops to roughly one third. For the two-airline/16 flight scenario, the probability of an equitable solution is approximately 10%

Property #7 Increasing the number of groups (e.g. airlines), decreases the probability of an equitable solution

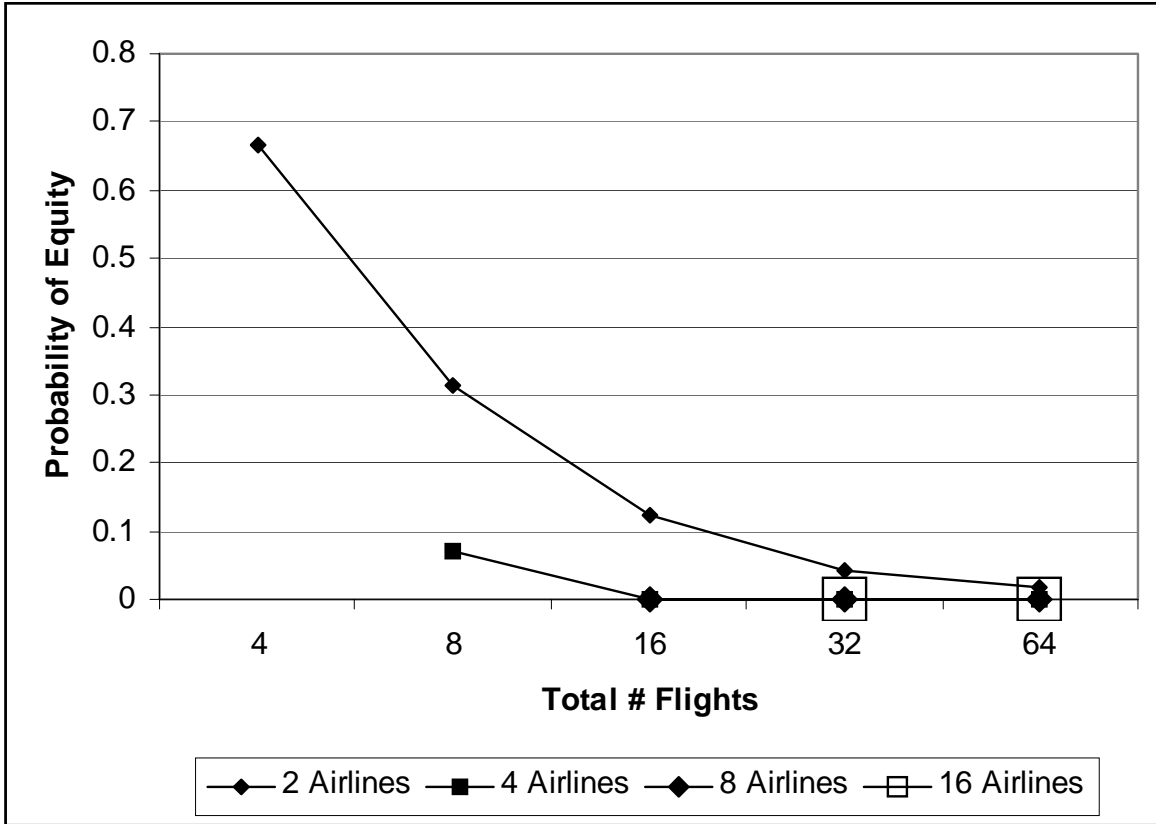


Figure 6: Results of simulation experiments to estimate the probabilities of proportional allocation of slots. Each data point is estimated using one million random slot allocations. The x-axis represents the number of flights scheduled and the y-axis shows the probability of a proportionally equitable allocation.

As the number of groups (e.g. airlines) increases, the probability of an equitable allocation decreases. This property is also a function of the number of permutations and the decreased likelihood of randomly scheduling an equitable allocation.

The probability decreases extremely fast in the number of airlines. Since *every* airline must have the same total delay, there are more constraints on the situation when there are more airlines. For example, if 3 airlines are involved and each airline has 4 slots, the probability of proportional equity is less than 1%. If 4 airlines are involved, the value drops to about .01%. If 5 or more airlines are involved, the probability is less than one in a million (no schedules with proportional equity were observed in the simulations). In fact, the only cases in which a “reasonable” probability of a proportional equity is observed are cases involving 2 airlines, and this is still typically below 10% and closer to 1%.

V. Conclusions and Future Work

When a centralized entity is responsible for allocation of overscheduled resources, the degrees of authority to allocate the flights with proportional equity are limited by the symmetry of schedules generated by the individual groups.

The low probability of achieving equity suggests that the notion of equity needs to be reconsidered. alternate approaches:

- 1) Market-based mechanisms: Slot assignment using this approach ensures an economically efficient use of the resource. Criticism of this approach is that societal needs can be ignored and require regulatory intervention. One example is Essential Air Services program, that subsidizes air service to rural markets that cannot sustain service by profit-seeking airlines that operate in a de-regulated marketplace. See (Ref 10, 11).

- 2) Refining First-Scheduled/First-Allocated to Improve Equity: TFM initiatives are designed to “exempt” flights from delay allocations (i.e. these flights maintain their original scheduled slot) and “compress,” “swap” and “substitute” flights to improve their position in the allocation. These adjustments provide mechanisms to adjust the allocation thereby adjusting the equity. Future work is required to establish the degree to which these adjustments can be made. For example, asymmetrical compression may balance out equity.
- 3) Temporal Measures of Equity: Instead of measuring equity over a single allocation, equity could be measured across multiple allocations (e.g. days). Given the degree of accuracy in which flights can be controlled to arrive at the constrained resource, the equity may even out over time. The degree to which normal arrival errors adjust affect equity is a subject for future work.
- 4) Variable Allocation Schemes: Combined with temporal measures of equity, variable allocation schemes may result in more equitable allocations.
- 5) Hybrid Allocation Schemes: A market-based allocation scheme with regulatory constraints may yield the most economically efficient and socially equitable allocation. Equity measured simultaneously on multiple dimensions may yield more equitable system-wide equity. See Schaar & Sherry (Ref 8).
- 6) Realistic Equity Boundaries The underlying “structure” of fleet mix and route structures bounds the state-space in which the equity trade-offs can be made. These structures should be established and serve as the basis for equitable allocations.

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