QUANTITATIVE ESTIMATION OF WAKE VORTEX SAFETY USING THE P2P MODEL

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Abstract

Wake vortex encounter risk is a major safety issue in aircraft final approach phase. The evaluation of the risk is crucial when the industry takes the effort to reduce aircraft spacing for the purpose of capacity gain. This paper proposes a hybrid analytical method to assess the wake vortex encounter risk during the final approach phase. The hybrid method uses probabilistic method to calculate the risk based on the probabilistic distributions of aircraft positions and vortex characteristics obtained from Monte Carlo simulations. The proposed method will be more computational efficient than pure simulations to evaluate the probability of a rare event, such as a serious vortex encounter. In addition, it is able to provide insights about risk distributions with respect to time and location. The estimation given by the model is conservative, which provides effective safety margin to the risk evaluation in a highly stochastic environment.

Introduction

Wake vortices are a natural by-product of aircraft lift. A lighter weight aircraft encountering the wake vortex of a heavier aircraft will suffer a roll upset and may lose control.

Research on airplane wake vortex physics, characterization, forecasting, and related aircraft separation has been ongoing both in Europe and the United States. With the aid of large, fast computers to use Large Eddy Simulation (LES) models, researchers have much better understood the physics of vortices [1].

The position and strength of vortices are highly subject to aircraft characteristics and meteorological conditions. Under certain situations, vortices can remain in flight corridors for long periods of time. To avoid wake vortex encounters, following aircraft must maintain a safe distance from the leading aircraft to ensure adequate time for the vortices to decay. The Federal Aviation Administration of the USA and the International Civil Aviation Organization have divided airplanes into several weight classes and established safe separation in the terminal approach area for each pair of airplane categories. Air traffic controllers must abide by the wake vortex separation rules to space approach aircraft under Instrument Meteorological Conditions (IMC).

These separation standards are believed to be one of major constraints to aviation system capacity [2][3]. Hinton et al. [4] estimate that an average capacity gain of about 12% can be expected by reducing wake vortex separation behind heavy aircraft by approximately 1.3 NM and reducing spacing behind B757 by 1.2 NM. Although Gerz et al. [2] do not believe such gains are realistic, they do think modified wake vortex separation can lead to tactical and strategic improvements. They project that even the increase of a few slots per day at a busy airport would be of great value.

Kos and Blom (et al. [5]) warn that wake vortex induced risk should be better understood before new ATM concepts, including the reduction of separation, for departure and landing on busy airports are deployed. Based on the risk assessment methodology proposed by Blom et al. in [6], Kos and Blom (et al. [5]) have introduced a probabilistic methodology to evaluate wake vortex induced risk.

The aim of this paper is to demonstrate a new method to analyze the risk of wake vortex encounters during the final approach phase. The paper is organized as follows. The next section will briefly describe the numerical representation of vortex characteristics and will emphasize a particular vortex model, the Probabilistic 2-Phased (P2P) model. The third section will discuss the existent method to evaluate wake vortex safety, and present a new analytical method to assess the probability of a wake vortex encounter. An illustration of the proposed method follows, and the paper ends with a discussion about the assumptions of the method and future work.

Numerical Description of Wake Vortices and the P2P Model

Because only vortices in the far field are of safety interest to us, we focus on the numerical description of a vortex that has rolled up. The initial vortex distance b_0 is proportional to the wingspan B, and the proportional constant is the span-wise load factor, s. That is, $b_0 = s\, B$.

The age t of a vortex is usually normalized to obtain a non-dimensional time, $t^* = t/t_0$, so that results from various flight stages, even CFD (Computational Fluid Dynamics) simulations can be compared. The reference time t_0 is defined as the time it takes for a vortex to propagate one initial vortex distance downward. Mathematically,

$$t_0 = 2\pi s^2 B^2 / \Gamma_0.$$

The actual circulation Γ of a vortex is normalized by a reference circulation, $\Gamma^* = \Gamma/\Gamma_0$, where

$$\Gamma_0 = Mg/(\rho\, s\, B\, V).$$

The descent speed is also normalized: $\omega^* = \omega/\omega_0$., where the reference descent speed is $\omega_0 = \Gamma_0/(2\pi b_0)$.

Based on the definitions of those basic parameters, various mathematical models have been developed to describe the decay and evolution process of vortices. Detailed review of common models can be found in [1], [2], [7].

Different from most of the mathematical wake vortex models, the P2P (Probabilistic 2-Phase decay model) proposed by Holzapfel ([8][9][10]) is able to accommodate the stochastic nature of turbulence, complex vortex instabilities and uncertainties of environmental parameters by giving bounds to a deterministic prediction according to corresponding confidence levels. The P2P model depicts the evolution process of a wake vortex in two phases, as suggested in LES (Large Eddy Simulation) data. One phase is a diffusion phase, the other is a fast decay phase. The decay parameters are calibrated to calculated LES prediction and observed wake vortex data.

The P2P model uses the average circulation, $\Gamma_{5-15,}$ over vortex circles with radii from 5 to 15 meters. The advantages of using average circulation as a metric to describe vortex strength are low sensitivity to observation angles, automatic compensation of vortex motion, and smoothing of the variance. The values of circulation in the diffusion phase and the fast decay phase are given in equation (3) and (5) in [8]. The onset time of fast decay and decay rate depend on meteorological parameters, ε, the eddy dissipation rate, and N, the Brunt-Vaisala frequency.

The P2P takes into account that the turbulence deforms and transports in a stochastic way and results in considerable variation in strength and positions by giving upper bound and lower bounds of vortex positions and strength. To obtain the bounds, two runs of the P2P with different decay onset time and kinematic viscosity are conducted; then a constant uncertainty allowance of 0.2 Γ_0^* is added to (subtracted from) the initial circulation, and an uncertainty allowance of one initial vortex spacing is employed for lateral and vertical positions. An example of circulation prediction with upper/lower bounds based on the P2P model is shown in Figure 1.

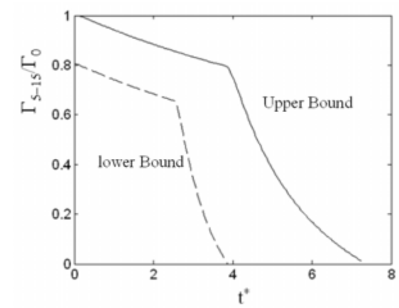

Figure 1. Prediction of Vortex Circulation with Bounds Based on the P2P Model.

From a safety perspective, the upper bounds of vortex strength and position are of more interest because they are more conservative.

Existing Wake Vortex Safety Analysis

Modeling of wake vortex evolution is one step in analyzing encounter risk. To quantify the various levels of safety related with vortices, researchers must also integrate aircraft dynamics, and evaluate the probability of encounter with wakes of different strength. In addition, information of induced roll moment and compensation against roll should be used to evaluate the severity of encounters. Further evaluation even should include pilot's response at the moment of encounter.

NLR proposed a probabilistic methodology and developed a corresponding toolbox to assess vortex-induced risk ([5][6]). The probabilistic methodology first evaluates the wake vortex encounter probability based on the Monte Carlo simulations of aircraft dynamics and wake vortex evolutions; then it identifies the seriousness of encounters considering vortex strength and the trailing aircraft's rolling controllability. WAVIR (WAke Vortex Induced Risk) is the toolbox developed to integrate the simulation models. To estimate the wake vortex encounter probability, the probability density distributions (PDF's) of aircraft positions and vortex positions and strengths are obtained from the aircraft model and the vortex model. PDF's are inputted in another Monte Carlo simulation to count encounters.

A statistically significant estimation of such a rare event as wake vortex encounter from pure Monte Carlo simulations may be very computationally expensive. We introduce below a mathematical method to evaluate a conservative probability of a vortex encounter based on the P2P model. The method provides another perspective to investigate the wake vortex risk by numerically assessing the encounter risk with vortices at various ages. Furthermore, the conservatism inherent in the probabilistic model and the P2P model will help to provide a safety margin.

A Conservative Probabilistic Model

Consider the final approach process described in Figure 2. Aircraft fly a desired 3^o glide slope to land and the altitude deviation from the desired path shrinks as aircraft get closer to the runway. Wake vortices generated by a leading aircraft will decay and transport for a certain period depending on the characteristics of aircraft and current meteorological conditions. If vortices stay in the flight corridor for enough time, it is possible that a following aircraft will encounter the vortices. To mathematically express the probabilistic event, we identify aircraft as they arrive in succession by the index j. Thus, if the leading aircraft is j, the following aircraft is j+1. We assume that aircraft j+1 can encounter a wake generated by aircraft j, but not by aircraft j-1 or earlier. For notational purposes, subscripts x, y, z refer to longitudinal (along the flight direction), lateral (along the wing direction), and altitudinal positions. Δt is a small time step. X denotes a certain point whose distance to the final approach fix is X. Similarly, we use T to denote a certain time point which is T time after the leading aircraft passes the point X.

Regarding a wake vortex generated by aircraft j with wake age T at the point X, we define $W_y(j, T, X)$ as the lateral position of a wake generated by aircraft j at the longitudinal position X at a time T after the aircraft has passed X, and $W_z(j, T, X)$ as the altitudinal position, and its circulation is $W_\Gamma(j, T, X)$. Correspondingly, $a_y(j, X)$ and $a_z(j, X)$ are the lateral position and the altitudinal position of aircraft j at the longitudinal position X, respectively. Similarly, $a_t(j+1, X)$ is defined as the period of time after aircraft j passed X that aircraft $j+1$ passes X.

We define $I(j, X, T)$ as an indication function which has the value 0 or 1. If the lateral and altitudinal positions of the following aircraft j+1 are close to the positions of wake vortex generated by aircraft j time T ago, the value of the indication function is 1, otherwise, it is 0. Mathematically,

$$I(j,X,T)=\begin{cases} 1 & \begin{array}{l}\text{when } \left|\mathrm{a}_y(j+1,X)-W_y(j,T,X)\right|\le \Delta y \\ \text{AND} \left|\mathrm{a}_z(j+1,X)-W_z(j,T,X)\right|\le \Delta z\end{array} \\ 0 & \text{Otherwise}\end{cases}$$

Then $I(j, X, T) = 1$ stands for the event that the following aircraft has an encounter with the wake vortex at age T at the point X. The probability of this event is denoted as $M(j, X, T) = \mathrm{p}(I(j, X, T) = 1)$.

Assuming that vortices' lateral drifts are independent of vertical descents, we have

$$\begin{aligned}&\mathrm{M}(j,X,T)=\\&\mathrm{p}(\left|\mathrm{a}_y(j+1,X)-W_y(j,T,X)\right|\le \Delta y, \left|\mathrm{a}_z(j+1,X)-W_z(j,T,X)\right|\le \Delta z)\\&=\mathrm{p}(\left|\mathrm{a}_y(j+1,X)-W_y(j,T,X)\right|\le \Delta y)\\&\quad\times \mathrm{p}(\left|\mathrm{a}_z(j+1,X)-W_z(j,T,X)\right|\le \Delta z)\end{aligned}$$

The equation means that the probability that the trailing aircraft (j+1) encounters a vortex at location x generated by the previous aircraft (j) time T ago is the product of the probability that the trailing aircraft's latitude is within vortex's latitude range and the probability that the trailing aircraft's altitude is within vortex's altitude range.

Likewise, the indication function $I(j, X)$ is defined as

$$I(j,X)=\begin{cases}1 & \text{when } I(j,X,t)=1, \text{for any } t\in[T_1,T_2]\\ 0 & \text{Otherwise}\end{cases}$$

, where T_1, T_2 are the lower and upper bound of wake vortex age of interest. $I(j, X) = 1$ stands for the event that the following aircraft j+1 encounters a wake vortex at the point X which is generated by aircraft j. The probability is described as $M(j, X) = \mathrm{p}(I(j, X) = 1)$.

Furthermore, the indication function $I(j)$ is

$$I(j)=\begin{cases}1 & \text{when } I(j,X)=1, \text{for any } X\in[X_1,X_2]\\ 0 & \text{Otherwise}\end{cases},$$

where X_1, X_2 are the lower and upper bound of the final approach in consideration. $I(j) = 1$ means the event that the following aircraft encounters wake vortex generated by its previous aircraft along the final approach path. The probability is then $M(j) = \mathrm{p}(I(j) = 1)$.

For any event A, its probability is $\mathrm{p(A)} = \mathrm{p(AB)} + \mathrm{p(A\overline{B})}$, where B and $\overline{B}$ are exclusive events. Therefore, to evaluate $M(j, X, T) = \mathrm{p}(I(j, X, T) = 1)$**,** the probability can be written as

$$
\begin{aligned}
\mathrm{p}(I(j, X, T) = 1) &= \mathrm{p}(I(j, X, T) = 1, |a_t(j+1, X) - T| \le \frac{\Delta t}{2}, W_\Gamma(j,T,X) > 0) \\
&+ \mathrm{p}(I(j, X, T) = 1, |a_t(j+1, X) - T| > \frac{\Delta t}{2}, W_\Gamma(j,T,X) > 0) \\
&+ \mathrm{p}(I(j, X, T) = 1, |a_t(j+1, X) - T| \le \frac{\Delta t}{2}, W_\Gamma(j,T,X) \le 0) \\
&+ \mathrm{p}(I(j, X, T) = 1, |a_t(j+1, X) - T| > \frac{\Delta t}{2}, W_\Gamma(j,T,X) \le 0)
\end{aligned}
\tag{1}
$$

where $|a_t(j+1, X) - T| \le \frac{\Delta t}{2}$ means that when time T after aircraft j passes X, aircraft $j+1$ is passing X with a time difference smaller than Δt, and $W_\Gamma(j,T,X) > 0$ means that when time T after aircraft j passes X, the wake vortex generated at X has a circulation larger than 0.

It is obvious that the probabilities on the right side of (1) except for the first item are zeros, so assuming that $W_\Gamma(j,T,X) > 0$ and $|a_t(j+1, X) - T| \le \Delta t/2$ are independent,

$$
\begin{aligned}
&\mathrm{p}(I(j, X, T) = 1) = \\
&\mathrm{p}(I(j, X, T) = 1, |a_t(j+1, X) - T| \le \frac{\Delta t}{2}, W_\Gamma(j,T,X) > 0) \\
&= \mathrm{p}(I(j, X, T) = 1 \,|\, W_\Gamma(j,T,X) > 0, |a_t(j+1, X) - T| \le \frac{\Delta t}{2}) \times \\
&\mathrm{p}(W_\Gamma(j,T,X) > 0) \times \mathrm{p}(|a_t(j+1, X) - T| \le \frac{\Delta t}{2})
\end{aligned}
\tag{2}
$$

We then formulate the probability of an encounter of aircraft $j+1$ at longitude X with a vortex at any age. When vortices at age between T_0 and T_n are concerned, the probability of an encounter is

$$
\begin{aligned}
&p(I(j, X) = 1) = p(I(j, X, T_0) = 1) \cdot \frac{\Delta t'}{\Delta t} + \\
&p(I(j, X, T_1) = 1) \cdot \frac{\Delta t'}{\Delta t} + \ldots + p(I(j, X, T_n) = 1) \cdot \frac{\Delta t'}{\Delta t}
\end{aligned}
$$

where $I(j, X, T_0) = 1$, $I(j, X, T_1) = 1$, … , $I(j, X, T_n) = 1$ are mutually exclusive events. $\Delta t'$ is the sampling time step, and $\Delta t' \ge \Delta t$. We assume $p(I(j, X, T)=1)$ to be uniformly distributed in the interval $[T-\Delta t/2, T+\Delta t/2]$. Let $M(j, X) = \mathrm{p}(I(j, X) = 1)$, since $M(j, X, T) = \mathrm{p}(I(j, X, T) = 1)$, then if $\Delta t'$ and Δt are small enough,

$$
M(j, X) = \sum_{i=0}^{n} p(I(j, X, T_i) = 1) * \frac{\Delta t'}{\Delta t} \approx \int_{T_0}^{T_n} \frac{M(j, X, T)}{\Delta t} dT
\tag{3}
$$

Because $M(j, X, T)$ is actually a probability over Δt, when we divide it by Δt, $M(j, x, T)/\Delta t$ is a probability density function. Since the strengths of vortices are strongly correlated with vortex ages, the probability of encountering vortices at various ages can be used to represent the probability of encountering vortices with various strengths. For example, vortices generated by a large aircraft at an age younger than 70 seconds may be viewed as young vortices, and those older than 70 seconds may be viewed as old. The encounters with young vortices are generally more dangerous than those with old vortices.

Similarly, let $\Delta x'$ is the sampling distance step, and when $\Delta x'$ and Δx are small enough, the probability that aircraft $j+1$ hits vortices with their age between $[T_0, T_n]$ generated by aircraft j over the final approach path between $[X_0, X_m]$ is

$$
\begin{aligned}
&M(j) = p(I(j, X_0) \cup I(j, X_1) \cup \ldots \cup I(j, X_m)) \le \\
&\sum_{i}^{m} p(I(j, X_i)) \cdot \Delta x = \sum_{i=0}^{m} M(j, X_i) \Delta x \approx \\
&\int_{X_0}^{X_m} M(j, x) dx = U(j)
\end{aligned}
\tag{4}
$$

Equality is achieved in the first line when the events $I(j, x_i)=1$ are mutually exclusive. Then $U(j)$ is an upper bound of the probability of a following aircraft encountering vortices at age between T_0 and T_n generated by its previous aircraft over the final approach path. Because a vortex encounter at a certain time and a certain location is a rare event, we expect that the bound will not be too loose.

To summarize the calculation steps of the wake vortex encounter risk, we first need to calculate $M(j, X, T) = p(I(j, X, T)=1)$, for each $X \in [X_0, X_m]$ and $T \in [T_0, T_n]$; then calculate $M(j, X)$ for each X, and finally obtain the value of $M(X)$.

An Illustration of the Conservative Probabilistic Model

For the purpose of evaluating wake vortex risk, we must integrate a wake vortex evolution model with an aircraft kinematic model. Because we need to consider the position distributions of both aircraft and wake vortex resulting from the variation of aircraft speeds, weights, wingspans, separations, and meteorological fluctuation, both the vortex model and aircraft model should be stochastic.

Aircraft Evolution at Final Approach

Without losing generality, we first study the case that both the leading aircraft and the following one are large aircraft, Boeing 727, Boeing 737 or MD80, etc. The characteristics of the aircraft are list in Table 1. We assume all the three characteristics follow Normal distributions with means and standard deviations given in Table 1. The data in Table 1 are used only for the illustration of the model. They are a ballpark estimation and not obtained from the collection of real data. The mean and the standard deviation are chosen to ensure that the maximum landing weight will not be exceeded. For instance, the maximum landing weight for a B727-100 is 64638 kg. The probability of a random number drawn from *Normal*(50000, 2000^2) larger than 64638 is around 10^{-13}. The standard deviation of weight in Table 1 is difficult to estimate because it depends on the load of passengers and luggage.

Table 1. Hypothetical Aircraft Characteristics

	Weight (kg)	Wingspan (meter)	Speed (mtr/sec.)
Mean	50,000	30	70
Std.Dev	2,000	2	4

In this paper, we assume that each aircraft flies a constant, but randomly chosen, speed throughout the final approach. After determining the aircraft speed, the time it takes an airplane to fly from the final approach fix to a certain point in the approach path can be calculated. Let $F_{X\ j}$ be the time of aircraft j to fly from the final approach fix to X, $F_{X\ j} = X/v$, where v is the flight speed. We assume that v follows a Normal distribution. Although this technically implies that $F_{X,\ j}$ mathematically is not normally distributed, since the values of v are far enough from zero, $F_{X,\ j}$ can be approximated using a Normal distribution. Figure 3 shows the distributions of flight time it takes an airplane to fly to the locations 4000 meters and 7000 meters away from the final approach fix respectively, as well as their Normal distribution fit.

The variance of the flight time to a further point from the final approach fix is bigger with no controller's interference. The increased variance will make the distribution of separation between two aircraft flatter.

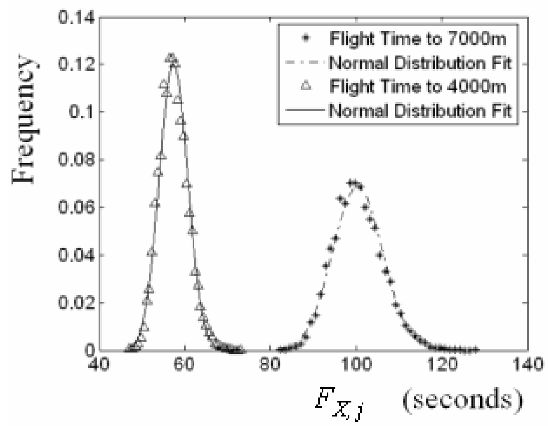

Figure 2. Flight Time to a Certain Point

Define T_j as the time of aircraft j passing the final approach gate, T_j^* as the time passing a certain point X, then the separation at X is $T^*_{j+1} - T^*_j = (T_{j+1} - T_1) + (F_{X,\ j+1} - F_{X,\ j})$. $F_{X,\ j+1}$ and $F_{X,\ j}$ follow the same distribution if they are the same type. The expected value $E[T^*_{j+1} - T^*_j] = E[T_{j+1} - T_j]$, and the variance $Var[T^*_{j+1} - T^*_j] = Var[T_{j+1} - T_j] + 2*Var[F_{X,j}]$.

For instance, if the separation at the final approach fix between two large aircraft is 64 seconds with a standard deviation of 5 seconds, the average flight time to the point with longitude of 7000 meters is 100 seconds, and its standard deviation is 6 seconds. Then $T^*_{j+1} - T^*_j$ can be approximated using the distribution *Normal*(64, 5^2+2*6^2). Now the probability of a following aircraft passing the point at 7000 meters at the moment t seconds after the leading one passes it can be calculated.

$$p(|a_t(j+1, X) - T| \le \frac{\Delta t}{2}) =$$

$$p((T_2^* - T_1^*) < T + \frac{\Delta t}{2}) - p((T_2^* - T_1^*) < T - \frac{\Delta t}{2})$$

where Δt is a small time step.

If $t=90$, $\Delta t=2$,

$$p(|a_t(j+1,\ 7000) - 90| \le \frac{2}{2}) =$$

$$p((T_2^* - T_1^*) < 91) - p((T_2^* - T_1^*) < 89) = 0.0025$$

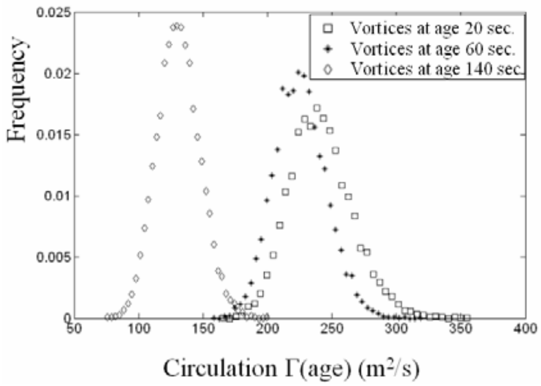

Figure 3. Circulations of Vortices at Different Ages

Wake Vortex Evolution at Final Approach

We continue to use a homogeneous mix of all large aircraft to illustrate wake vortex evolution at the final approach phase. The major characteristics of aircraft are shown in Table 1, and the hypothetical values of the meteorological parameters are listed in Table 2. The variables are assumed to follow Normal distributions. Generally, given a certain level of turbulence represented by eddy dissipation rate (or stratification represented by Brunt-Vaisala frequency), the lower the stratification, the longer the lifespan of a vortex. The values listed in Table 2 are relatively small with respect to stratification and background turbulence, which result in longer lifespan of vortices. A more detailed discussion on the effects of these parameters on wake vortices can be found in [10]. The wake vortex age in consideration is from 0 to 200 seconds.

Table 2. Hypothetical Meteorological Parameters

	Air Density (kg/m^3)	**Brunt–Vaisala Frequency**	**Eddy Dissipation Rate**
Mean	1.0	0.016	$3.2*10^{-5}$
Std.Dev	0.1	0.002	10^{-6}

In this paper, we only consider the evolution of wake vortices on the altitude dimension, and assume that we can ignore the drifting distance of vortices on the lateral dimension. Based on the values of aircraft characteristics and meteorological conditions listed in Table 1 and Table 2, we computed values on vortex circulation and descent distance at various ages from Monte Carlo simulations, based on the P2P model. The histograms of example computation results at age 20 seconds, 60 seconds, and 140 seconds are shown in Figure 3 and Figure 4 for the upper bounds of circulation and descent distances respectively. Unless explicitly stated, the circulations or positions calculated from the P2P model that we are using later on in the paper are upper bounds.

Figure 4 shows that older vortices have a lower average value and but higher variance for circulation strength than younger vortices. The phenomenon is reasonable since normalized circulation $\Gamma^*(\tau)$ reduces with the increment of vortex age τ.

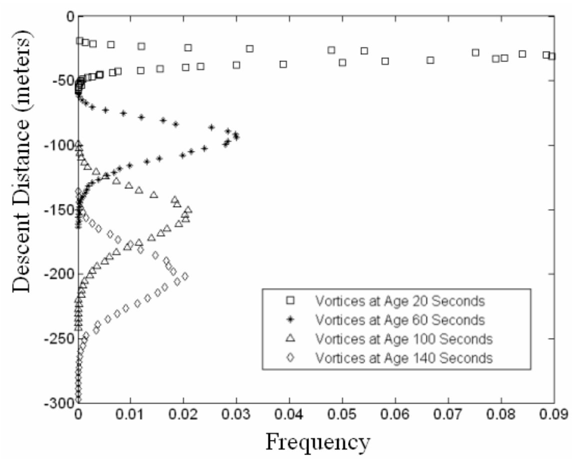

Figure 4. Descent Distances of Vortices at Different Ages

The distributions of vortices' descent distance have larger spread when vortices get older. For the same reason with the situation of vortex's circulation described above, the variances of vortex's descent speed reduce as age increases, but it leads to larger variance of distance because distance is the integral of speed over time, as shown in Figure 4.

When aircraft fly closer to the runway threshold, the altitude deviations of their flight path will be diminishing. Our aircraft dynamics model reflects the change of deviation and the examples of the histograms of the aircraft altitudes and the upper bounds of wake vortex altitudes are shown in Figure 5.

In Figure 5, the altitude distribution of a following aircraft at a certain location (e.g. 2000 meters from the final approach fix) is independent of the longitudinal separation with the leading aircraft. So the distribution of (t = 20 seconds) is overlapped with that of (t=100 seconds). Old vortices descend more than young ones with larger variances, as illustrated by Figure 4. If we define a vortex encounter as the event that the following aircraft flies under the upper bound of vortex altitudes, the point that we can see from Figure 5 is that, at a certain location, a following aircraft is more likely to hit younger vortices if it passes by, given the configured meteorological conditions. For example, the conditional probability of hitting vortices at age 20 seconds is 0.032, while that of hitting vortices at age 100 seconds is $6*10^{-14}$. Here we assume that the probability of a trailing aircraft flying under the lower bound of vortices is ignorable.

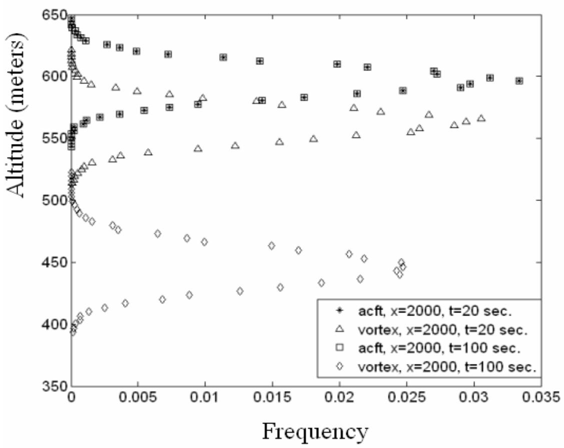

Figure 5. Aircraft Altitude Histograms

The probability of aircraft passing by when a vortex at age 20 or 100 seconds have to be considered to evaluate the real encounter probability. Because the desired separation for a large-large flight mix is 2.5 NM, the time separation is around 64 seconds assuming the flight speed is 140 knots. From the discussion above about the aircraft evolution model, $p(|a_t(j+1, 2000) - 20| \le \Delta t/2) = 2\times10^{-37}$, which makes the overall probability of hitting a vortex at age 20 seconds is very small, actually it is $2\times10^{-37}\times0.032\approx6\times10^{-39}$.

Because the uncertainty of flight time increases with the distance to the runway threshold decreasing, as shown in Figure 3, the probability $p(|a_t(j+1, X) - T| \le \Delta t/2)$ displays an interesting feature with respect to the distance *x* and vortex age *t*, shown in Figure 6. According to the aircraft flight model given above, aircraft enter the final approach with separation standard, which is around 64 seconds. So the possibilities of aircraft passing at around 64 seconds are much higher than others. However, owing to the growing variance of flight time when aircraft approach to the runway, the differences among probabilities of aircraft passage are becoming smaller.

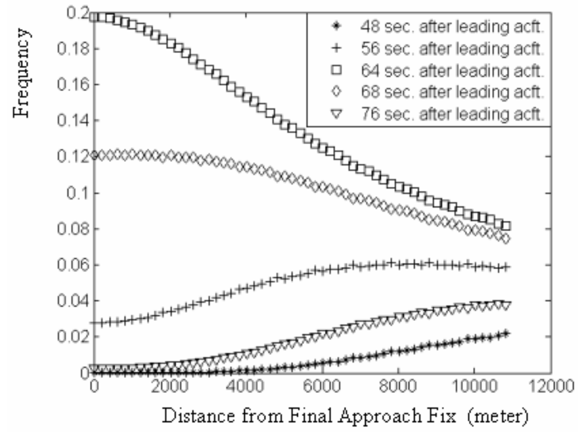

Figure 6. Probability of Aircraft Passing A Location at Different Time

The conditional probabilities of wake vortex encounter vanish along the longitude due to the reduction of the variance of the flight altitudes. The conditional encounters are less likely to happen to old wake vortices since they are further away from the glide slope than younger vortices. Figure 7 displays the examples of the vortex encounter probability conditioning that a following aircraft passes and vortex is alive.

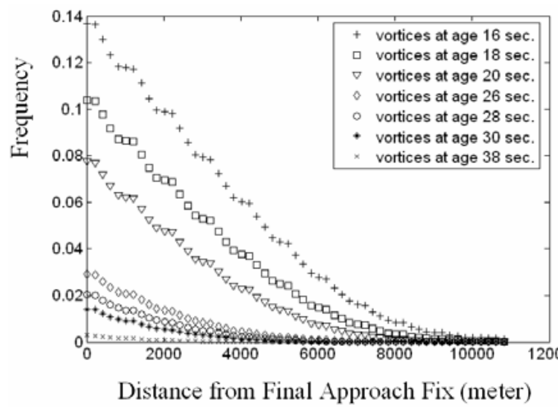

Figure 7. Conditional Probability of Vortex Encounter

The evolution of the probability *M*(*j, X, T*) over the longitude *X* and vortex age *T* is more complicated, and it combines all the factors we have discussed above. Figure 9 shows the curves of *M*(*j, X, T*) when *T* is much less than the assumed separation standard, 64 seconds; Figure 10 shows the curves of *M*(*j, X, T*) when *T* is close to 64 seconds. Because the value is a product of conditional encounter probability and aircraft passage probability, and when *T* is relatively small, i.e less than 50 seconds, conditional encounter probability is large and decreasing with respect to *X*, while aircraft passage probability is small and increasing with *X*. Therefore some of the curves in Figure 8 are not monotonically decreasing. When *T* is large enough, both the conditional encounter probability and the aircraft passage probability are decreasing with *X*, so the curves of *M*(*j, X, T*) in Figure 9 are monotonically decreasing.

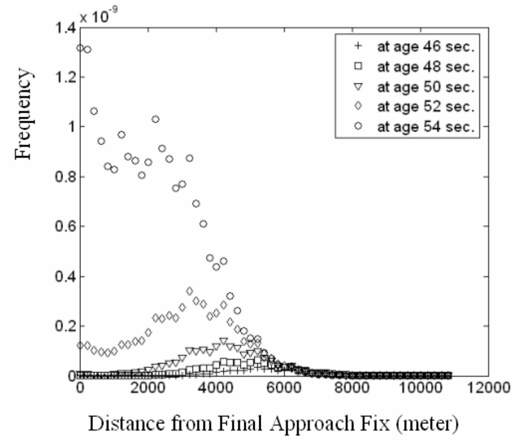

Figure 8. Probability of Vortex Encounter at a Location when Vortex Age is far From 64 Seconds

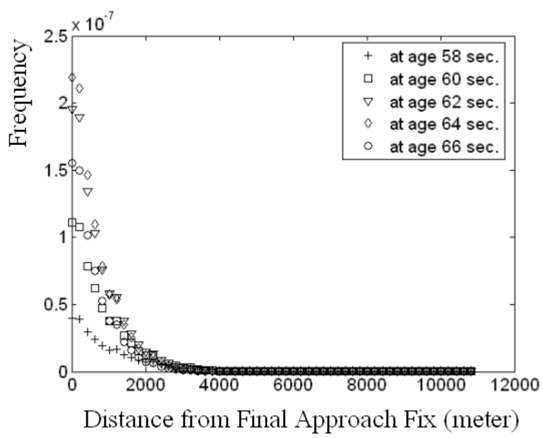

Figure 9. Probability of Vortex Encounter at a Location When Vortex Age is close to 64 Seconds

The integral of *M*(*j, X, T*) over the final approach path and over vortex ages is:

$$U = \int_{X_0}^{X_{RWY}} \int_0^{200} \frac{M(j,X,T)}{\Delta t} dtdx = 2.09 \times 10^{-4} .$$

The value of *U*(*j*) means that the probability of the following aircraft *j+1* hitting vortices generated by aircraft *j* should not be greater than $2.09*10^{-4}$.

If we are interested in the probability of hitting vortices at certain range of age, the value can be calculated by adjusting the range of *t* in the integral. For example, the probability of hitting vortices at ages younger than 40 seconds is about $2*10^{-7}$, and the marginal probabilities over the longitude is shown in Figure 11. The probability of hitting vortices at ages between 40 seconds and 70 seconds are $2.04*10^{-4}$, and the marginal probability distribution is shown in Figure 12. It demonstrates that most of the likely encounters are related with vortices at age around the deployed separation, and severe encounters are rather rare. According to Gerz et.al ([2]), about 80 encounters per year on average at London-Heathrow Airport. The amount of annual operations in 2003 is around 460,700 ([11]), so the estimated probability of a human-sensible encounter is about $1.7*10^{-4}$. Although we cannot use the Heathrow data to verify the vortex risk calculated from the conservative probabilistic model, it shows that our estimation is located in a reasonably correct region. A detailed and comprehensive verification needs carefully collected data not only about aircraft operations, but also the meteorological conditions.

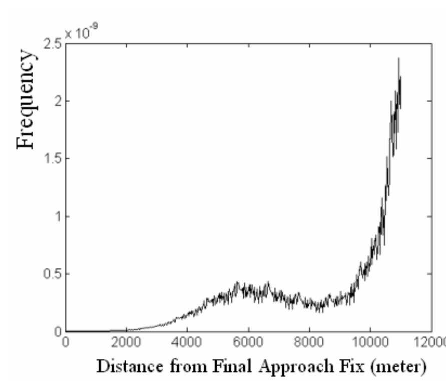

Figure 10. Probability of An Encounter with Vortices Younger than 40 Seconds

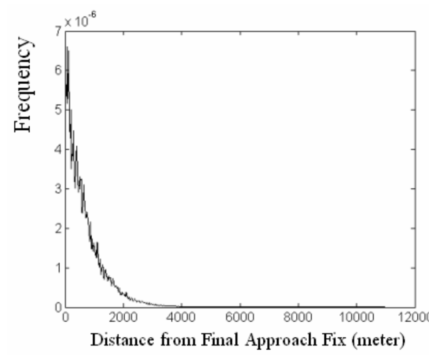

Figure 11. Probability of An Encounter with Vortices at Age Between 40 sec. and 70 sec

Discussion

The conservative probabilistic model proposed in this paper is based on a conservative model of wake vortex decay and transport, the P2P model. However, other numerical models of wake vortex are also applicable and the estimation of vortex risk is still conservative. Conservatism in risk evaluation is essential because the system involves uncertainties that are difficult to accurately identify or quantify, and a certain safety margin can help to reduce the impact of these unknown uncertainties.

The major concern about the model is that the estimation might be much larger than the true risk. This can be seen from Equation 3. When a part or all of events $I(j, X_i)$ are not mutually exclusive, $M(j)$ might be much less than $U(j)$. The upper bound would be close to the true risk only if events $I(j, X_i)$ are nearly mutually exclusive. As an alternate approach, Kos et al. [6] choose the maximum value of $p(I(j, x_i)=1)$ over x as the induced risk. This is an underestimate of the overall risk, and is accurate when the events $I(j, x_i)=1$ are highly dependent. Using this approach, for the example given in this paper, we get $Max(p(I(j, x_i)=1); x_i)=3.3*10^{-6}$, compared to the upper bound previously computed, $2.09*10^{-4}$.

One intermediate approach is to divide the approach path into segments of equal length, and to assume that encounters are dependent within an interval, but mutually exclusive between intervals. If we divide the flight distance into m sections with equal length, and define the kth section as X_k and $\mathrm{Max}_k(\mathrm{p(I(j, x_i))}; \mathrm{x_i} \in X_k)$ as the maximum probability of an encounter happening in the section X_k, then the overall probability is

$$\sum_{k=1}^{m} Max_k(p(I(j, x_i)); x_i \in X_k).$$

If we choose the length of a section to be 100 meters, the resulting probability is about $2.5*10^{-5}$, and if the length is 400 meters, the probability is $7*10^{-6}$. As we choose a larger length of a section, the result approaches Kos's estimation. As we choose a smaller length, the result approaches $U(j)$.

Basically the proposed conservative probabilistic model is a hybrid analytical model. The methodology is different from the pure simulation method given by [5], but it uses a numerical probabilistic method to calculate the risk while simulations are conducted to obtain probability distributions of aircraft positions and vortex characteristics. Numerical methods have the advantage of computational efficiency over pure simulations in evaluating the probability of a rare event, such as a serious wake vortex encounter. Another strength of the analytical model is that it provides more insights on risk distributions in terms of time and location than pure simulations. Care must be taken that the fitting distribution may have tails that are beyond the data generated by simulations.

Risk distributions depend highly on meteorological conditions, the aircraft flight model, and aircraft characteristics. Therefore the graphs and numerical results given in the paper are only notional. But, a calibrated analysis with carefully collected data can be conducted based on the method illustrated in the paper.

The physics of wake vortices can be much more complicated than that illustrated in this paper. For example, vortices may rebound in strongly stratified atmosphere or due to ground effect. In such situations, the analytical result of encounter risk can be very different with that given in this paper.

Although the probabilistic model does not give risk evaluation directly related with vortex strength (circulation) but via vortex ages, it is easier to understand and use from the perspective of operations. In risk classification analysis, risk categories should be based on vortex ages, while circulation information will not be lost in that vortex circulation is highly coupled with vortex age.

In this paper, we assume there is no crosswind, so the drift in the lateral direction is negligible. This assumption is conservative because aircraft and wakes are always at the same positions in the lateral dimension, so there is no possibility of avoiding a wake due to lateral drift. Taking into account the lateral drift will reduce the upper bound of estimation, especially when a crosswind is present. An analysis with crosswind using the conservative probabilistic model has been carried out, and will be reported in a future paper, which will also include the sensitivity analysis of wake vortex encounter risk to crosswind and separation.

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