Airline Scheduling Optimization
(Chapter 7 – I)

Vivek Kumar
(Research Associate, CATSR/GMU)

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CENTER FOR AIR TRANSPORTATION SYSTEMS RESEARCH

GEORGE MASON UNIVERSITY

Agenda

- Airline Scheduling
  - Factors affecting decision
  - Complexity and challenges
- Airline Schedule Planning Overview
  - Fleet Assignment Problem
    - Greedy Solution/Shortcomings/Need for Time-Space network
    - Fleet Assignment Mode
      - Basic FAM
      - Shortcomings of BasicFAM, Spill cost/recapture..
      - Extended FAM
      - IFAM (Itinerary Based)
  - Schedule Design Optimization
  - Crew and Maintenance Optimization Preview..

Objective of this Class

Objective of this Class

Table 7.1 Example: flight schedule, fares and passenger demands

Flight no.	From	To	Dep. time (EST)	Arr. time (EST)	Fare ($)	Demand (passengers)
CL301	LGA	BOS	11:00	12:00	150	250
CL302	LGA	BOS	12:00	13:00	150	250
CL303	LGA	BOS	14:00	15:00	150	100
CL331	BOS	LGA	08:00	09:00	150	150
CL332	BOS	LGA	11:30	12:30	150	300
CL333	BOS	LGA	14:00	15:00	150	150
CL501	LGA	ORD	12:00	15:00	400	150
CL502	LGA	ORD	13:00	16:00	400	200
CL551	ORD	LGA	08:00	11:00	400	200
CL552	ORD	LGA	09:30	12:30	400	150

Assign Fleet Types to Each Leg using Optimization to Maximize Profit.

Output of "Schedule Design"

Factors affecting Airline Scheduling Decision (MACRO level)

- Market Demand (all PAX not same),
- Fleet composition,
- Location of crews,
- Maintenance bases,
  - $7.5 million last March against SWA. 46 B737 jets on 59,791 flights in 2006 and 2007 without mandatory fuselage inspections for fatigue cracking. Six planes had cracks, the FAA says. After SWA became aware it hadn't made the inspections, the airline continued to operate the 46 planes on an additional 1,451 flights.
- Gate restrictions,
- Landing slot restrictions (eg: NY airports),
- For International flights: bilateral agreements

Complexity of the Problem is affected by…

- Airports are not similar
  - Arr/Dep restrictions, Gates (type/personnel), Equipments..
- Fleet composition
  - Different operating characteristics, costs, maintenance and crew requirements, seating capacity …
- Crews
  - Crews capable of operating only certain aircraft types, Limitations of when/how they can work…
- Different O-D markets
  - Different demand volume, profitability/customer demographics..

Airline Schedule Planning challenges..

- STOCHASTIC problem,
  - Uncertainty in PAX demand, Pricing of tickets, Fuel, Crew availability, Weather …
- SIZE of problem
  - Break into sub problems and proceed..

Airline Schedule Planning

Schedule Design

↓

Fleet Assignment

↓

Aircraft (Maintenance) Routing

↓

Crew Scheduling

Select optimal set of *flight legs* in a schedule
(Flight legs to operate: Origin, Sch Dep Time, Approx Arrival Time, Frequency)

Assign aircraft types to flight legs such that *contribution* is maximized

Contribution = Revenue - Costs

Assign crew (pilots and/or flight attendants) to flight legs

Each problem solved in order, with output of previous subproblem used as input for next subproblem

The Fleet Assignment Problem

- Outline
  - Problem Definition and Objective
  - Fleet Assignment Network Representation
  - Fleet Assignment Model

Problem Definition

<u>Given:</u>

- Flight Schedule
  - Each flight covered exactly once by one fleet type
- Number of Aircraft by Equipment Type
  - Can't assign more aircraft than are available, for each type
- Turn Times by Fleet Type at each Station
- Other Restrictions: Maintenance, Gate, Noise, Runway, etc. (Not addressed in formulation)
- Operating Costs, Spill and Recapture Costs, Total Potential Revenue of Flights, by Fleet Type

Problem Objective

Find:

- Cost minimizing (or profit maximizing) assignment of aircraft fleets to scheduled flights such that *maintenance* requirements are satisfied, conservation of *flow* (balance) of aircraft is achieved, and the number of aircraft used does not exceed *inventory* (in each fleet type)

Table 7.1

Table 7.1 Example: flight schedule, fares and passenger demands

Flight no.	From	To	Dep. time (EST)	Arr. time (EST)	Fare ($)	Demand (passengers)
CL301	LGA	BOS	11:00	12:00	150	250
CL302	LGA	BOS	12:00	13:00	150	250
CL303	LGA	BOS	14:00	15:00	150	100
CL331	BOS	LGA	08:00	09:00	150	150
CL332	BOS	LGA	11:30	12:30	150	300
CL333	BOS	LGA	14:00	15:00	150	150
CL501	LGA	ORD	12:00	15:00	400	150
CL502	LGA	ORD	13:00	16:00	400	200
CL551	ORD	LGA	08:00	11:00	400	200
CL552	ORD	LGA	09:30	12:30	400	150

Output of "Schedule Design"

"Market"

Figure 7.1 and Table 7.2

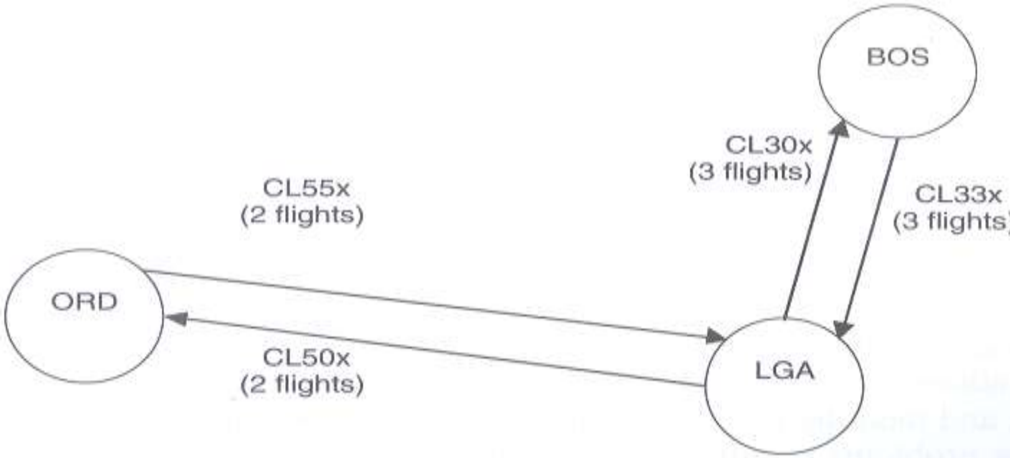


Figure 7.1 Example flight network

Table 7.2 Fleet information

Fleet type	No. of aircraft owned	Capacity (seats)	Per flight operating cost ($000)	
			LGA–BOS	LGA–ORD
DC9	1	120	10	15
B737	2	150	12	17
A300	2	250	15	20

$$c_{l,f} = fare_l \times \min(D_l, Cap_f) - OpCost_{l,f}$$

with:

$c_{l,f}$: profitability of assigning fleet type f to flight leg l;
$fare_l$: fare of flight leg l;
D_l: demand of flight leg l;
Cap_f: capacity of fleet type f;
$OpCost_{l,f}$: operating cost of assigning fleet type f to flight leg l.

Profit Calculation

Table 7.3 Profit ($000 per day)

Flight no.	DC9	B737	A300
CL301	8	10.5	22.5★
CL302	8	10.5	22.5★
CL303	5★	3	0
CL331	8	10.5★	7.5
CL332	8	10.5	22.5★
CL333	8	10.5★	7.5
CL501	33	43★	40
CL502	33	43	60★
CL551	33	43	60★
CL552	33	43★	40

LGA – BOS

Fare: 150

Demand : 250

Capacity(B737): 150

Operating Cost of B737 on LGA-BOS route: 12K

150*min(250,150) – 12k

= 10.5k

Greedy Approach

Greedy Solution and Shortcoming

- Static Network Representation is INSUFFICIENT to capture the ‘temporal nature’.
  - Solution is a Time-Space Network..

Figure 7.2

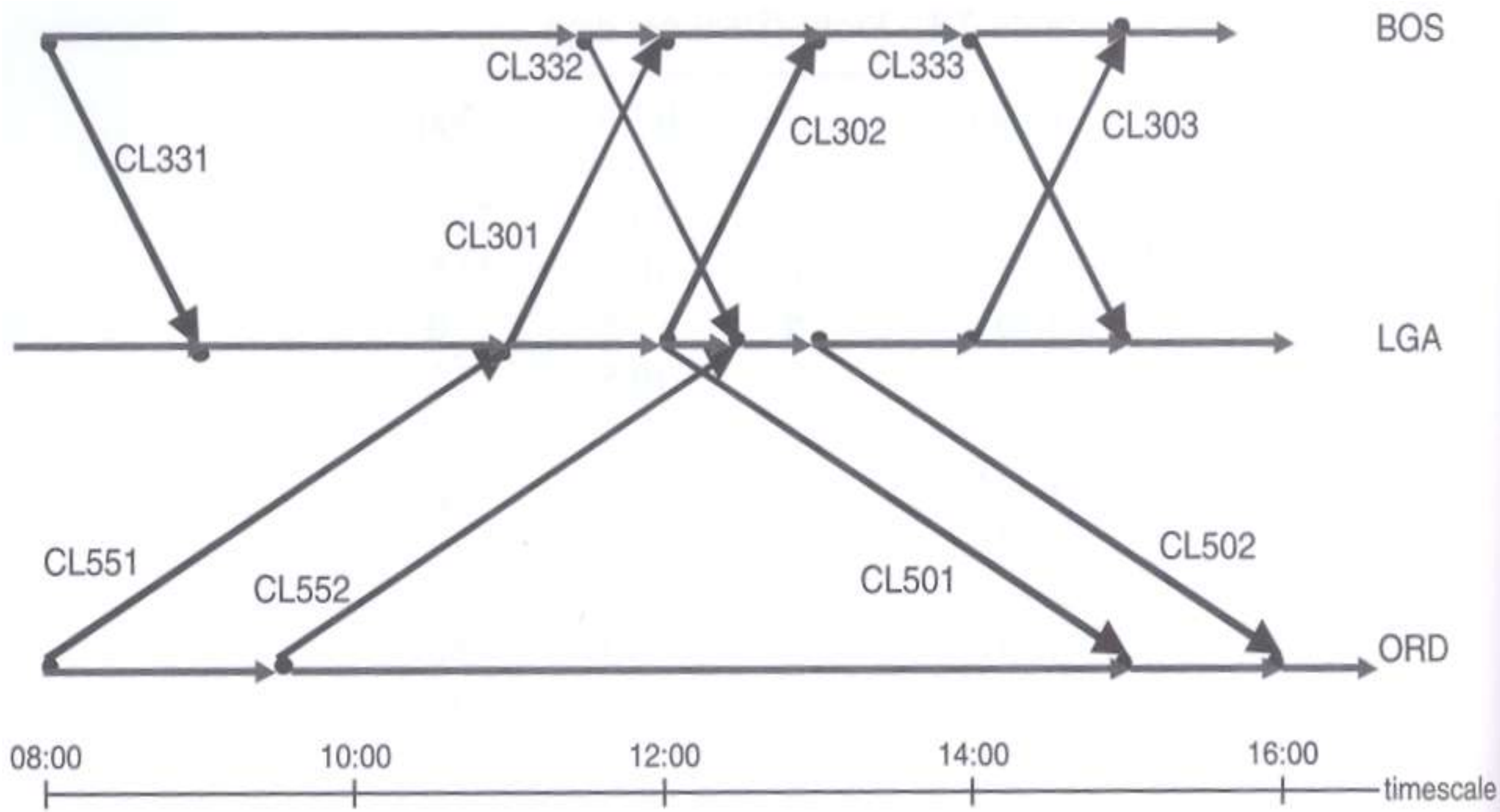


Figure 7.2 Time–space network

Figure 7.3

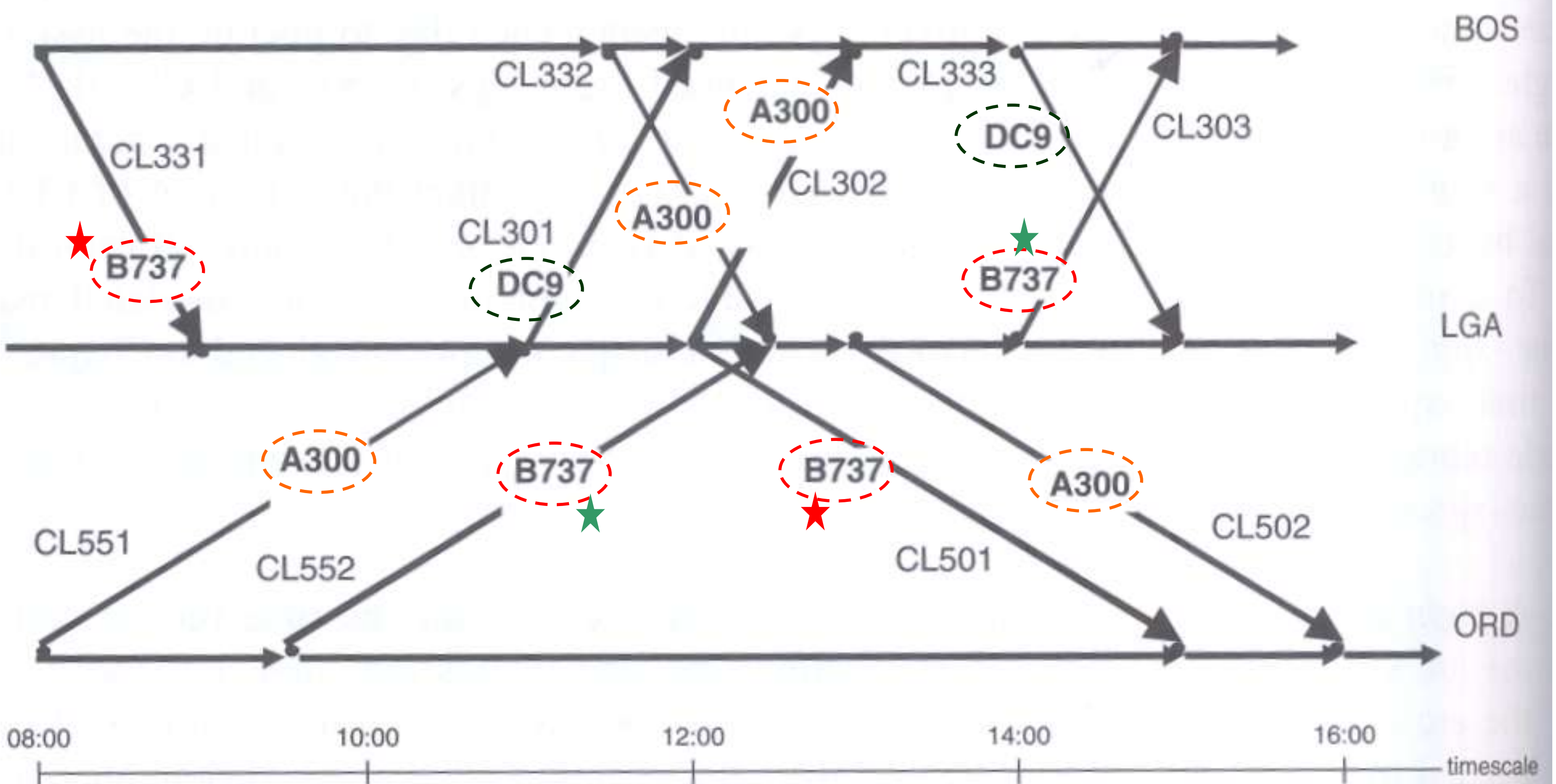


Figure 7.3 Optimal fleet assignment

A300's end up at different locations. Profit: 280,500

Figure 7.4

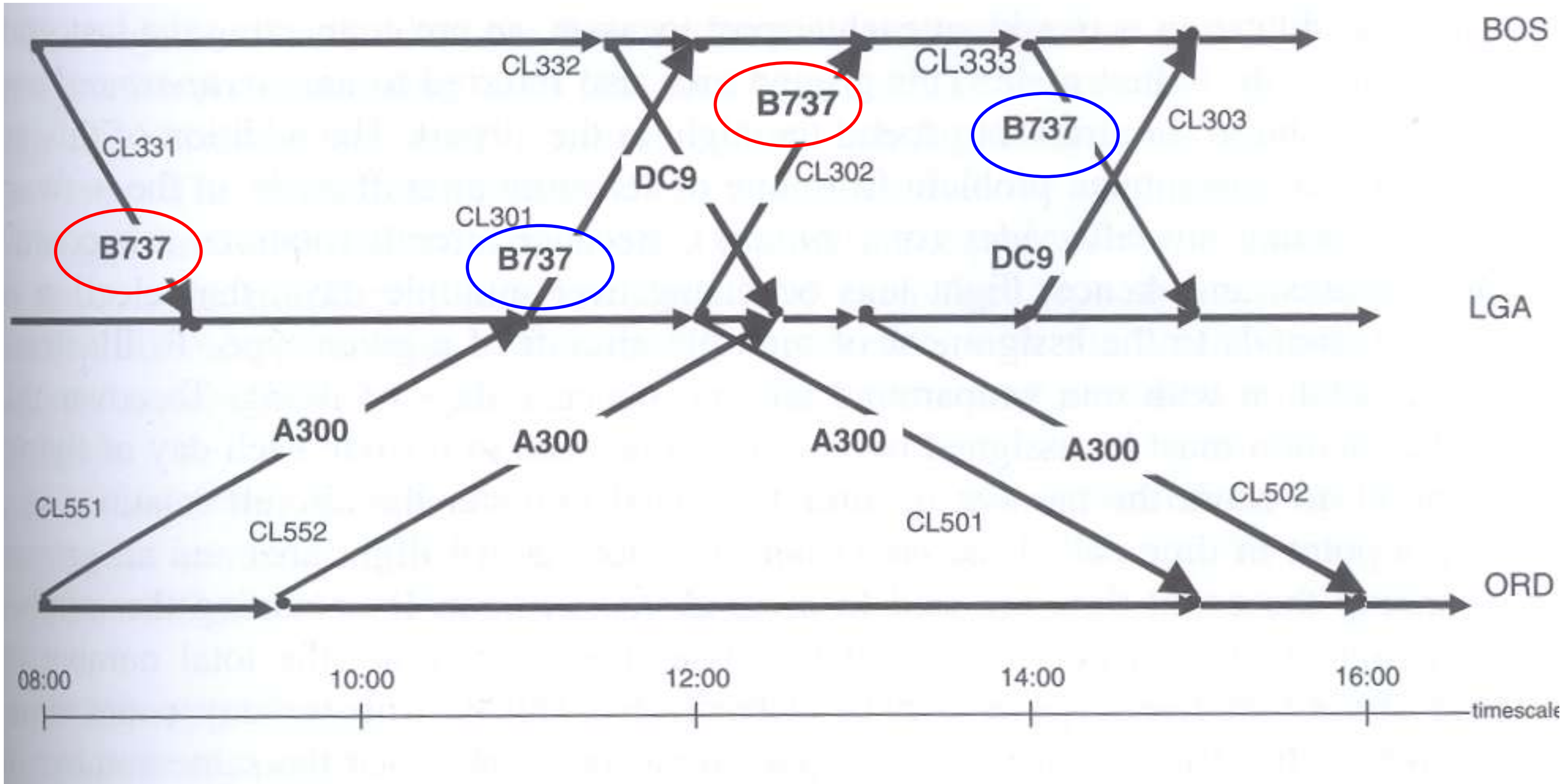


Figure 7.4 Suboptimal fleet assignment

A300's end up at same location. Profit: 255,000

Time-Line Network

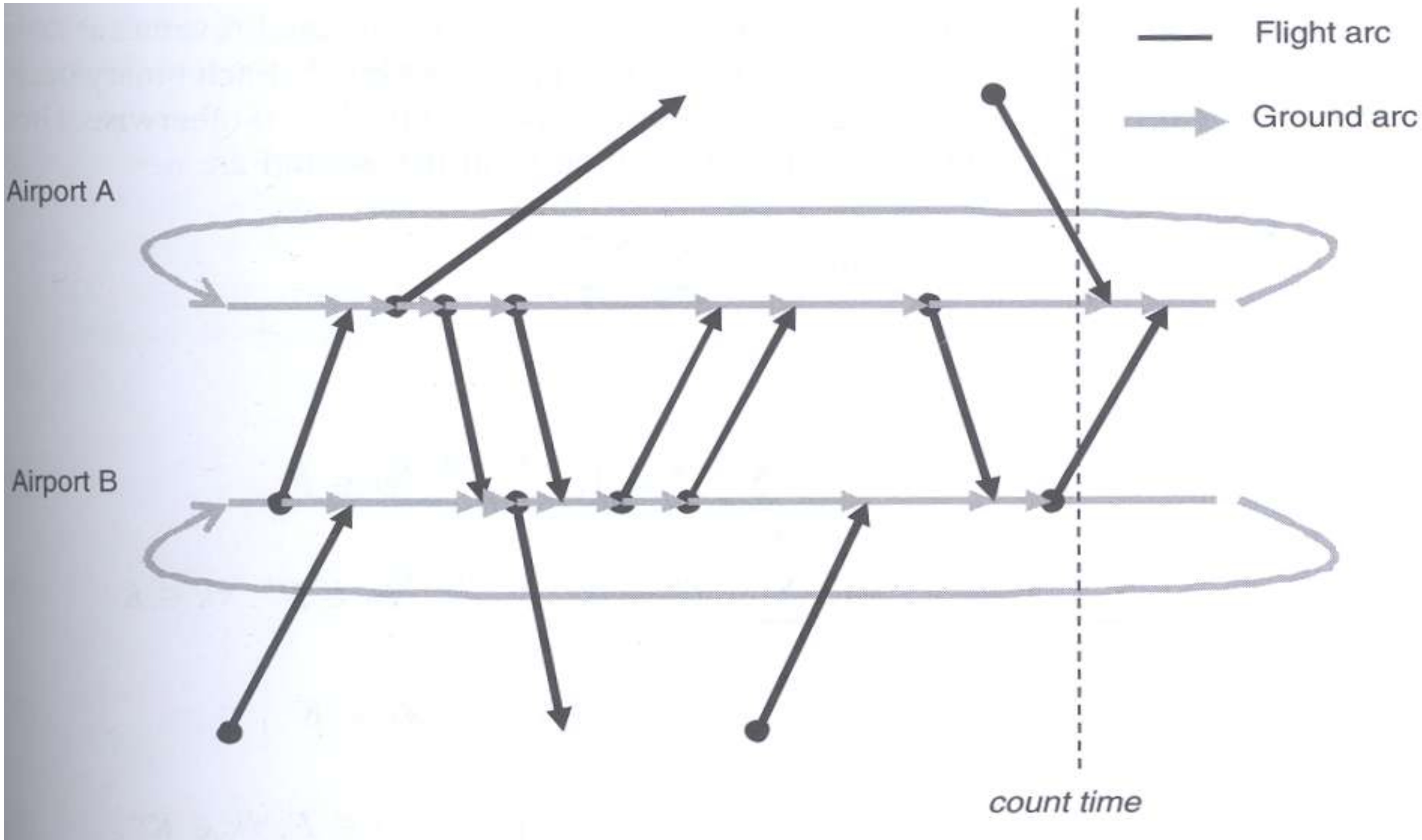


Figure 7.5 Fleet-specific time–space network with count time and wraparound ground arcs

Basic FAM

$$\text{Minimize} \sum_{i \in F} \sum_{k \in K} c_i^k f_i^k$$

subject to:

Serve All flight legs with exactly 1 fleet type

$$\sum_{k \in K} f_i^k = 1, \qquad \forall i \in F \qquad (7.1)$$

Balance at each Airport

$$y_{n^+}^k + \sum_{i \in O(k,n)} f_i^k - y_{n^-}^k - \sum_{i \in I(k,n)} f_i^k = 0, \qquad \forall n \in N^k, \forall k \in K \qquad (7.2)$$

Don't exceed availability for each fleet type

$$\sum_{a \in CG(k)} y_a^k + \sum_{i \in CL(k)} f_i^k \leq M^k, \qquad \forall k \in K \qquad (7.3)$$

$$f_i^k \in \{0, 1\}, \qquad \forall i \in F, \forall k \in K \qquad (7.4)$$

$$y_a^k \geq 0, \qquad \forall a \in G^k, \forall k \in K \qquad (7.5)$$

Legend:

n^- : ground arc terminating at node n
n^+ : ground arc originating at node n

f^i_k = 1, leg i serviced by fleet k,

y^k_a = # of acft of type k on ground arc a

M^k = # of aircrafts of fleet type k available

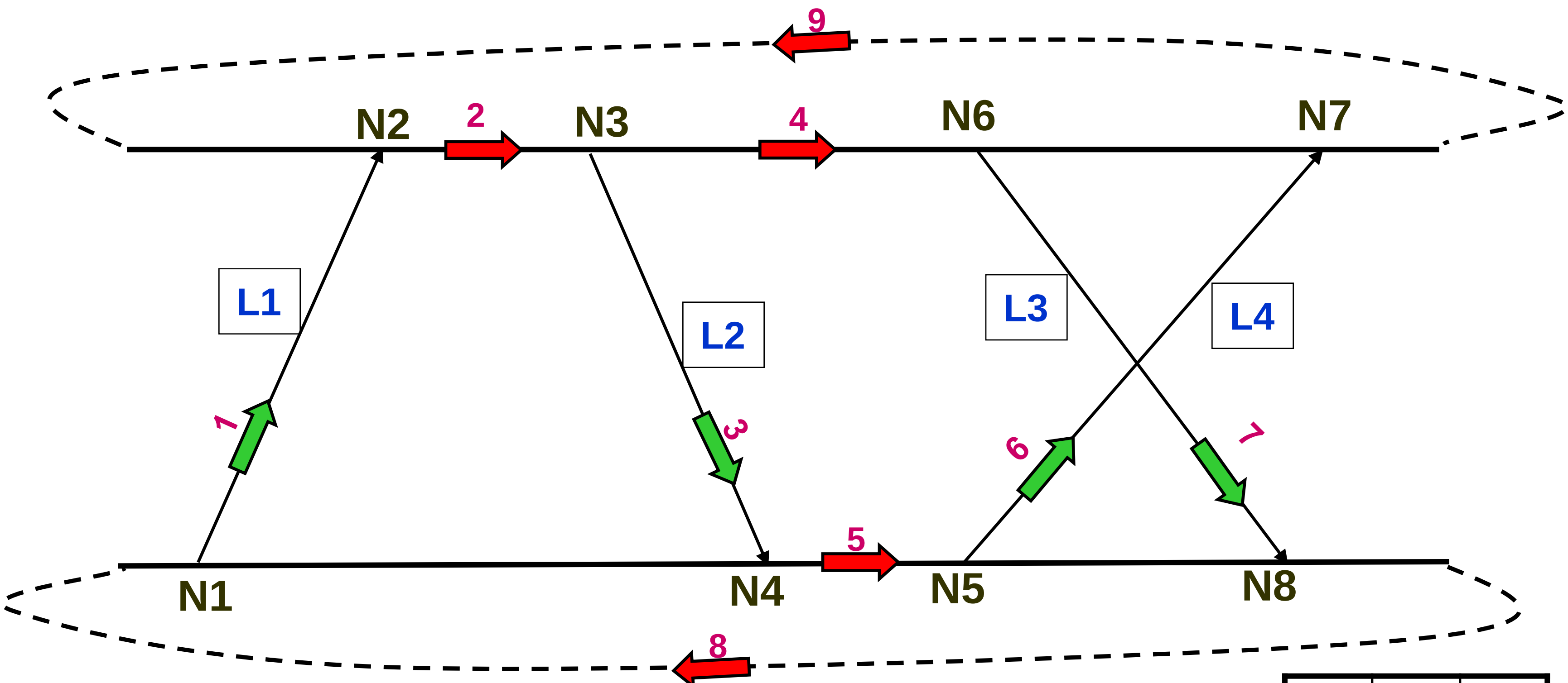
N^k = Set of nodes for fleet k

G^k = set of ground arcs for fleet k

O(k,n) and **I(k,n)** = set of flights ***originating*** and ***terminating*** at node n in fleet k's time-space network

CL(k) and **CG(k)** = set of flight legs and Ground Arcs that cross the count time in fleet k's network

Example



Node	+	-
N1	Ø	8
N2	2	9
N3	4	2
N4	5	Ø
N5	Ø	5
N6	Ø	4
N7	9	Ø
N8	8	Ø

Nodes = {N1,N2,N3,N4,N5,N6,N7,N8}   Arcs = {1,2,3,4,5,6,7,8,9}

Ground Arcs = {2,4,5,8,9}   Flight Arcs = {1,3,6,7}

i = {L1, L2, L3, L4}

k = {1,2 } ----- {B757, DC90}

$M^1 = M^2 = 2$

$N^1 = N^2$ = {N1,N2,N3,N4,N5,N6,N7,N8}

$G^1 = G^2$ = {2,4,5,8,9}

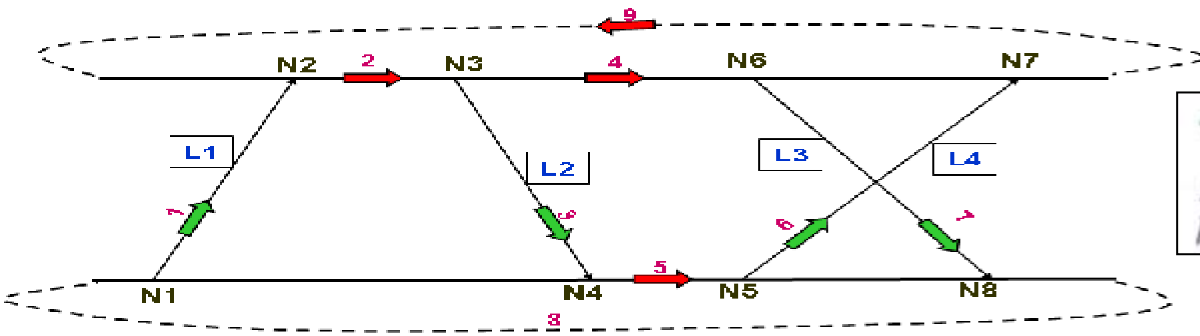
O(1,N1) = L1 , O(1,N3) = L2 , O(1,N5) = L3 , O(1,N6) = L4, O(1, N2|N4|N7|N8) = null (Same for k = 2)

I(1,N2) = L1 , I(1,N4) = L2 , I(1,N8) = L3 , I(1,N7) = L4, I(1, N1|N3|N5|N6) = null (Same for k = 2)

CG(1) = CG(2) = {8,9}

CL(1)= CL(2) = Ø

Serve All Flight Legs (7.1)



$$\sum_{k \in K} f_i^k = 1, \qquad \forall i \in F$$

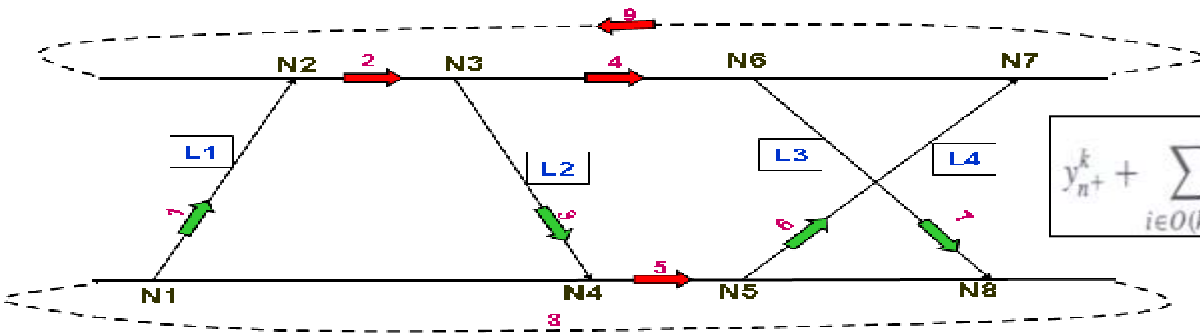
$$f_{i=L1}^{k=1} + f_{i=L1}^{k=2} = 1$$

$$f_{i=L2}^{k=1} + f_{i=L2}^{k=2} = 1$$

$$f_{i=L2}^{k=1} + f_{i=L2}^{k=2} = 1$$

$$f_{i=L2}^{k=1} + f_{i=L2}^{k=2} = 1$$

Balance Constraint (7.2)



$$y^k_{n^+} + \sum_{i\in O(k,n)} f^k_i - y^k_{n^-} - \sum_{i\in I(k,n)} f^k_i = 0, \qquad \forall n \in N^k, \forall k \in K$$

n=N1
i=1

$$y^{k=1}_{a=N1^+} + \sum_{i\in O(1,N1)} f_i^{k=1} - y^{k=1}_{a=N1^-} + \sum_{i\in I(1,N1)} f_i^{k=1} = 0$$

ø | L1 | 8 | ø

0	$f^{k=1}_{i=L1}$	$y^{k=1}_{a=8}$	0

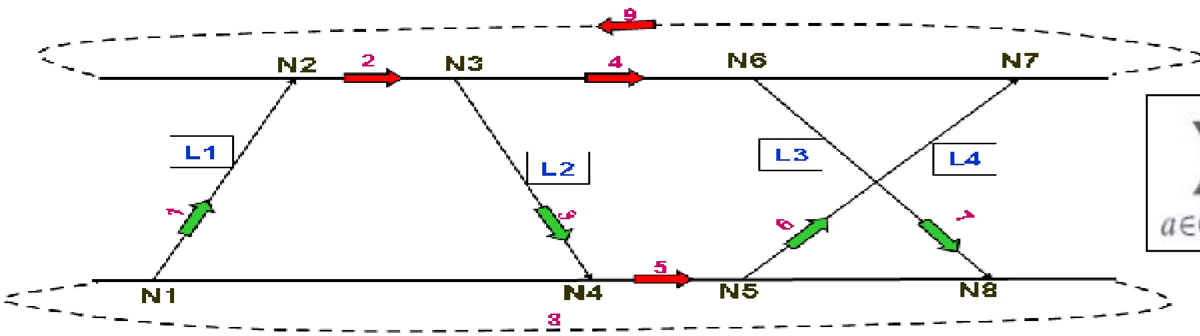
n=N1
i=1

$$y^{k=1}_{a=N4^+} + \sum_{i\in O(1,N4)} f_i^{k=1} - y^{k=1}_{a=N4^-} + \sum_{i\in I(1,N4)} f_i^{k=1} = 0$$

5 | ø | ø | L2

$y^{k=1}_{a=5}$	0	0	$f^{k=1}_{i=L2}$

Count Constraint (7.3)



$$\sum_{a \in CG(k)} y_a^k + \sum_{i \in CL(k)} f_i^k \le M^k, \qquad \forall k \in K$$

$$\sum_{a \in CG(k=1)} y_a^{k=1} + \sum_{i \in CL(k=1)} f_i^{k=1} \le M^{k=1}$$

8,9   ø

$$y_{a=8}^{k=1} + y_{a=9}^{k=1} \qquad 0 \qquad 2$$

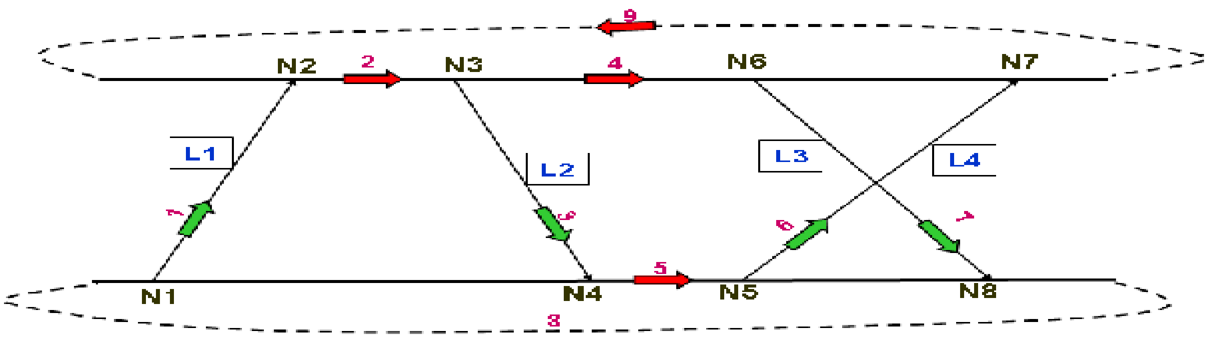
Legend:

CL(k) and **CG(k)** = set of flight legs and Ground Arcs that cross the count time in fleet k's network

CG(1) = CG(2) = {8,9}

CL(1)= CL(2) = Ø

Number of Variables



$$f_i^k \in \{0, 1\}, \qquad \forall i \in F, \forall k \in K \qquad (7.4)$$

$$y_a^k \geq 0, \qquad \forall a \in G^k, \forall k \in K \qquad (7.5)$$

i = {L1, L2, L3, L4}
k = {1,2 }
G1 = G2 = {2,4,5,8,9}

i(4) * k(2) = 8 ; **f Binary**

a(5) * k(2) = 10 ; **y (automatically Integer because of balance and non-negativity constraints)**

10+8 = 18 variables

FAM can be augmented with..

- Noise Restriction constraints
- Maintenance requirements
- Gate restrictions
- Crew considerations

Solution Time

- Table 7.4

Shortcoming of FAM

- Spill Cost and Recaptures ignored
- Consider only aggregate demand and average fares.
- Static demand is assumed (no seasonality etc considered)

Extending FAM : Introduction to Spilling

Example

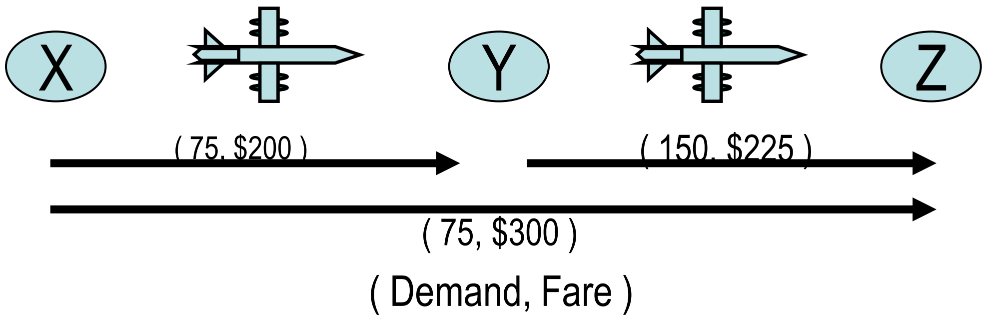


Table 7.5 Demand data

Market	Itinerary (sequence of flights)	Number of passengers	Average fare
X–Y	1	75	$200
Y–Z	2	150	$225
X–Z	1–2	75	$300

Max Possible Revenue

= 75*200 + 150*225 + 75*300

= 71,250

Table 7.6 Seating capacity

Fleet type	Number of seats
A	100
B	200

Table 7.7 Operating costs

Fleet type	Flight 1	Flight 2
A	$10 000	$20 000
B	$20 000	$39 500

Table 7. Possible fleeting configurations

Fleeting	Flight 1	Flight 2	Total operating cost	
I	A	A	$30 000	**10+20**
II	A	B	$49 500	**10+39.5**
III	B	A	$40 000	**20+20**
IV	B	B	$59 500	**20+39.5**

Spilling

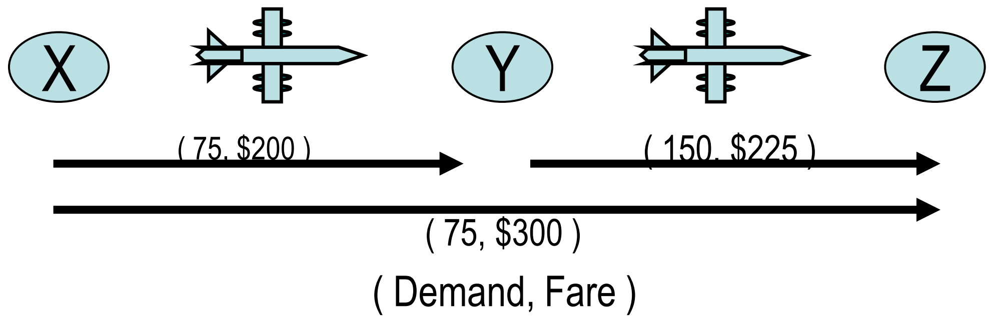
- FAM is leg-based
- Fares/PAX demand is itinerary (O-D pair) based

  Itinerary can be multiple legs. Leading to mismatch.

- Problem: Estimate “leg-bases Spill Costs”
  - Different methods:
    - Prorate total itinerary fare to flight legs s.t. their Sum equals total fare
      - Proration is typical done based on distance . Can also be done based on profitability, i.e. $/miles etc
      - Can also assign entire itinerary fare to each leg. Rationale: PAX will travel ALL or NO legs for any given itinerary

- Assumption: Airline has full discretion in determining which passenger it wishes to accommodate.

Revenue Maximizing Strategy for Spilling



- If Fleeting I is selected, i.e. Aircraft type A on both legs.
  - Available seats on each leg = 100
  - Demand in X-Y leg = 75 (from X-Y) + 75 (from X-Z) = 150
  - Demand in Y-Z leg = 150 (from Y-Z) + 75 (from X-Z) = 225
  - Need to spill 50 (150-100) and 125(225-100) PAX from leg 1 and 2 respectively
  - X-Z Fare (300) < X-Y Fare(200) + Y-Z Fare(225)
    - Spill 50 X-Z PAX first
    - X-Y leg is not beyond capacity now
    - As Fare Y-Z < Fare X-Z, spill (225-50-100) Y-Z PAX

Result Using Revenue Maximizing Strategy

Table 7.9 Minimum spill costs and resulting contributions for each fleeting combination

Fleeting	Operating costs	Spilled passengers	Spill costs	Contribution
I	$30 000	50 X–Z, 75 Y–Z	$31 875	$9375
II	$49 500	25 X–Z, 25 X–Y	$12 500	$9250
III	$40 000	125 Y–Z	$28 125	$3125
IV	$59 500	25 Y–Z	$5 625	$6125

I: Contribution = Max Possible Revenue – (Spill + Operating Cost)
= 71250 – ((50*300 + 75*225)+ 31875)
= 9375

Minimize Spill Cost for Each Flight Leg – Greedy Approach

Table 7.10 The contribution using a greedy algorithm

Fleeting	Operating costs	Spilled passengers	Spill costs	Contribution
I	$30 000	50 X–Y, 125 Y–Z	$38 125	$3125
II	$49 500	50 X–Y, 25 Y–Z	$15 625	$6125
III	$40 000	125 Y–Z	$28 125	$3125
IV	$59 500	25 Y–Z	$5 625	$6125

I: Contribution = Max Possible Revenue – (Spill + Operating Cost)
= 71250 – ((50*300 + 125*225)+ 31875)
= 3125

Need for Mathematical Models and Optimization Approaches..

- Enumeration of possible fleeting combinations for real scenarios is computationally expensive and sometimes even impossible.
  - AAL yielded annual improvement in revenue of .54 to .77%.

IFAM (Itinerary Based FAM) :

FAM with network effects

Expansion to basic FAM

- Include variables representing the mean number of PAX assigned to each itinerary in airline's network
  - t_p^r : Expected # of PAX desiring to travel on 'p' spilled to a different itinerary 'r'
- Recapture rate:
  - b_p^r : Estimated fraction of PAX spilled from 'p' and captured in itinerary 'r'
- Therefore,
  - $b_p^p = 1$: All PAX desiring to travel on p accept that itinerary
  - $b_p^r * \mathbf{t}_p^r$ = # of PAX traveling on 'r' that preferred 'p'

Itinerary-Based FAM (IFAM)

Kniker (1998)

Problem Formulation

Let P denote the set of itineraries, and let $\delta_i^r = 1$ if itinerary $r \in P$ contains flight leg $i \in F$, and $\delta_i^r = 0$ otherwise. These additional factors are added to the basic FAM to create the *itinerary-based fleet assignment model*, or IFAM, as follows:

$$\text{Minimize} \quad \sum_{i \in F} \sum_{k \in K} c_i^k f_i^k - \sum_{p \in P} \sum_{r \in P} fare_r b_p^r t_p^r$$

subject to:

$$\sum_{k \in K} f_i^k = 1, \qquad \forall i \in F$$

$$y_{n+}^k + \sum_{i \in O(k,n)} f_i^k - y_{n-}^k - \sum_{i \in I(k,n)} f_i^k = 0, \qquad \forall n \in N^k, \forall k \in K$$

$$\sum_{a \in CG(k)} y_a^k + \sum_{i \in CL(k)} f_i^k \leq M^k, \qquad \forall k \in K$$

$$\sum_{p \in P} \sum_{r \in P} \delta_i^r b_p^r t_p^r \leq \sum_{k \in K} CAP^k f_i, \qquad \forall i \in F \qquad (7.6)$$

$$\sum_{r \in P} t_p^r \leq D_p, \qquad \forall p \in P \qquad (7.7)$$

$$f_i^k \in \{0, 1\}, \qquad \forall i \in F, \forall k \in K$$

$$y_a^k \geq 0, \qquad \forall a \in G^k, \forall k \in K$$

$$t_p^r \geq 0, \qquad \forall p \in P, \forall r \in P \qquad (7.8)$$

IFAM Augmentations

$$\text{Minimize} \quad \sum_{i \in F} \sum_{k \in K} c_i^k f_i^k - \sum_{p \in P} \sum_{r \in P} fare_r b_p^r t_p^r$$

Operating Cost　　　Total Revenue

$$\sum_{p \in P} \sum_{r \in P} \delta_i^r b_p^r t_p^r \leq \sum_{k \in K} CAP^k f_i^k, \qquad \forall i \in F$$

Total # of PAX travelling on leg i

Max Capacity of the fleet type servicing flight leg i

$$\sum_{r \in P} t_p^r \leq D_p, \qquad \forall p \in P \qquad (7.7)$$

Unconstrained demand of P

Total # of PAX travelling on or spilled from itinerary p

Variables

IFAM vs FAM

Table 7.11 FAM vs. IFAM: problem size and solution times

Flight schedule: 2044 flight legs and 9 fleet types		
	FAM	IFAM
Problem size		
No. of columns	18 487	77 284
No. of rows	7 827	10 905
No. of non-zero entries	50 034	128 864
Solution time (seconds)	974	>100 000

Airline Schedule Planning

Schedule Design

Select optimal set of *flight legs* in a schedule

(Flight legs to operate: Origin, Sch Dep Time, Approx Arrival Time, Frequency)

Fleet Assignment

Assign aircraft types to flight legs such that *contribution* is maximized

Contribution = Revenue - Costs

Aircraft (Maintenance) Routing

Crew Scheduling

Assign crew (pilots and/or flight attendants) to flight legs

Each problem solved in order, with output of previous subproblem used as input for next subproblem

Schedule Design Optimization

- Data might not be available for Optimizing new schedule.
- Building new schedule from scratch may be computationally intractable.
- Dramatic changes to schedule not preferred as degree of consistency from one planning period to next, especially in business markets is highly valued.

Incremental Optimization

Also, not always possible to express ‘BEST’ schedule mathematically. (example..)

- Allow limited changes to a given/current schedule:
  - Airlines able to use historical booking data/traffic forecast
  - Required planning efforts and time manageable
  - Fixed investment at stations can be utilized efficiently (gate/aircraft lease agreements ..)
  - Consistency maintained for customers.

- Example: Retiming certain flight legs or replacing small set of unprofitable flight legs., redesigning airline hub connections...

Example : Hub Debanking

- Challenges posed:
  - Scheduling decision made for ALL flights legs, not just those at the hubs.
  - Fleeting decision renewed. Large/small example
  - Fleeting and Scheduling must be determined simultaneously. # of schedules is unlimited.

Optimizing Flight Retiming and Fleet Assignment Problem

- Special case of more generalized integrated schedule design and fleet assignment problem.
- Given: Set of flight legs to be operated
- Decision:
  - Flight retiming
  - Fleet Assignment
- Approach: In time-space network to include one flight arc copy for each possible departure time of each flight leg.

Formulation

$$\text{Minimize} \quad \sum_{i \in F} \sum_{k \in K} \sum_{b \in B^i} c_{i,b}^k f_{i,b}^k$$

subject to:

$$\sum_{k \in K} \sum_{b \in B^i} f_{i,b}^k = 1, \qquad \forall i \in F \qquad (7.9)$$

$$y_{n^+}^k + \sum_{(i,b) \in O(k,n)} f_{i,b}^k - y_{n^-}^k - \sum_{(i,b) \in I(k,n)} f_{i,b}^k = 0, \qquad \forall n \in N^k, \forall k \in K \qquad (7.10)$$

$$\sum_{a \in CG(k)} y_a^k + \sum_{(i,b) \in CL(k)} f_{i,b}^k \leq M^k, \qquad \forall k \in K \qquad (7.11)$$

$$f_{i,b}^k \in \{0, 1\}, \qquad \forall i \in F, \forall b \in B^i, \forall k \in K \qquad (7.12)$$

$$y_a^k \geq 0, \qquad \forall a \in G^k, \forall k \in K \qquad (7.13)$$

$f^k_{i,b} = 1$, if fleet type k is assigned to operate leg i and the departure time of leg I corresponds to the time of flight arc copy ‘b’

END Part I