

**What is risk of death (in a randomly selected flight)?**

**What is the risk of being involved in an accident (in a randomly selected flight)?**

**PRE-TEST**

1. You have a choice of 4 flights between an Origin (e.g. DCA) and a Destination (e.g. LAX). The flights use identical aircraft. Over the last 10 years of flights on this route, 1 in every 100,000 flights has resulted in an accident. Choose the safest flight in terms of risk of death, from one of the flights below:
  - a. Flight **A** flies a Great Circle Distance route between Origin and Destination. Load Factor on this flight is 50%
  - b. Flight **B** flies a Great Circle Distance route between Origin and Destination but a slower speed (to conserve fuel). Load Factor on this flight is 50%
  - c. Flight **C** flies a longer route (e.g. airway routes) than A and B, but flies at the (maximum) performance speed envelope and exhibits the same time enroute as Flight A.
  - d. All of the above
  
2. TRUE OR FALSE (CIRCLE ONE)
  1. Risk of death increases with the number of hours flown    T / F
  2. Risk of death increases when more passengers are on board    T / F
  3. Risk of death increases the longer the distance flown by each flight    T / F
  4. Risk of death is dependent on the “survivability technology” onboard airlines    T / F

## Aviation Safety Statistics

Fatal Event: Any circumstance where one or more passengers die during the flight from causes that are directly related to a civilian airline flight. These events include accidents, sabotage, hijacking, or military action.

Full Loss Equivalent (FLE) is the sum of the fractions of passengers killed for each fatal event divided by the total number of flights.

$$FLE = \sum x_i$$

where:

- $x_i$  is the fraction of passengers killed on each flight  $i$

For example, 50 out of 100 passengers killed is an FLE of 0.50, 1 of 100 would be a FLE of 0.01.

$$\text{Death Risk} = FLE/N$$

where:

- $N$  is the total number of flights

### Example for B757:

In 2000, the B757 had a total of five fatal events. In three of the five events, some passengers survived. The sum of the proportions of passengers killed was 3.4. Given the total of 8.7 million flights, that implied a fatal event rate of 0.39 per million flights. Note: when not counting survivability, the death-risk would be 0.57 per million flights.

<b>Metric</b>	<b>Source</b>	<b>Discussion</b>	
Fatal Accidents per 100,000 hours	NTSB <sup>1</sup>	Assumes accident risk increases with duration of flight Assumes all passenger experience fatalities	
Hull Losses per 100,000 flights	Boeing	<ul style="list-style-type: none"><li>• Use of departures recognizes that flight hours and flight distance do not correlate with accident risk</li><li>• Hull loss <math>\neq</math> fatalities</li></ul>	
Ratio: Passengers Killed to Passengers Carried		Vulnerable to fluctuations in Load Factor (i.e. more passengers on less flights vs more passengers on more flights)	
Fatal Accidents per 100,000 departures	NTSB <sup>1</sup>	<ul style="list-style-type: none"><li>• Use of departures recognizes that flight hours and flight distance do not correlate with accident risk</li></ul>	

<sup>1</sup> <http://www.nts.gov/aviation/Table1.htm>

NTSB

Table 1. Accidents, Fatalities, and Rates, 2008 Preliminary Statistics  
U.S. Aviation

	Accidents		Fatalities		Flight Hours	Departures	Accidents per 100,000 Flight Hours		Accidents per 100,000 Departures	
	All	Fatal	Total	Aboard			All	Fatal	All	Fatal
<b>U.S. air carriers operating under 14 CFR 121</b>	-	-	-	-	-	-	-	-	-	-
- Scheduled	20	0	0	0	18,730,000	10,597,000	0.107	-	0.189	-
- Nonscheduled	8	2	3	1	621,000	190,000	1.288	0.322	4.211	1.053
<b>U.S. air carriers operating under 14 CFR 135</b>	-	-	-	-	-	-	-	-	-	-
- Commuter	7	0	0	0	290,400	581,000	2.410	-	1.205	-
- On-Demand	56	19	66	66	3,673,000	-	1.52	0.52	-	-
<b>U.S. general aviation</b>	1,559	275	495	486	21,931,000	-	7.11	1.25	-	-
<b>U.S. civil aviation</b>	1,649	296	564	553	-	-	-	-	-	-
<b>Other accidents in the U.S.</b>	-	-	-	-	-	-	-	-	-	-
<b>- Foreign registered aircraft</b>	6	4	7	7	-	-	-	-	-	-
<b>- Unregistered aircraft</b>	7	1	1	1	-	-	-	-	-	-

Notes

Airsafe.com

<http://www.youtube.com/watch?v=WiFggLnji6c>

<b>Model</b>	<b>Death Risk</b>	<b>Flights</b>	<b><u>FLE*</u></b>	<b>Events</b>
<a href="#">Airbus A330</a>	UNK	UNK	1	1
<a href="#">Bombardier Dash 8</a>	UNK	UNK	2.38	4
<a href="#">BAe Jetstream**</a>	UNK	UNK	6.07	7
<a href="#">Dornier 228</a>	UNK	UNK	6.88	7
<a href="#">Dornier 328**</a>	UNK	UNK	0.11	1
<a href="#">Concorde***</a>	11.36	0.09M	1	1
<a href="#">Embraer Bandeirante**</a>	3.07	7.5M	23	28
<a href="#">Fokker F28**</a>	1.67	9.25M	15.45	21
<a href="#">Airbus A310**</a>	1.47	4.35M	6.38	8
<a href="#">Boeing 747</a>	0.74	18.47M	13.73	28
<a href="#">Embraer Brasilia</a>	0.71	7.4M	5.27	6
<a href="#">Boeing DC10**</a>	0.66	8.91M	5.91	15
<a href="#">Boeing 737-100/200**</a>	0.58	57.49M	33.53	47
<a href="#">Boeing DC9**</a>	0.57	62.33M	35.4	44
<a href="#">Airbus A300**</a>	0.53	11.35M	5.99	9
<a href="#">Boeing MD11**</a>	0.52	1.95M	1.02	3
<a href="#">Boeing 727**</a>	0.49	76.27M	37.2	48
<a href="#">Lockheed L1011**</a>	0.48	5.33M	2.54	5
<a href="#">BAe146/RJ100**</a>	0.47	9.62M	4.49	6

<a href="#">Boeing 767</a>	0.38	14.63M	5.5	6
<a href="#">Boeing 737 (all models)</a>	0.34	143.98M	49.48	68
<a href="#">ATR 42 and ATR 72</a>	0.33	13.2M	4.4	5
<a href="#">Boeing 757**</a>	0.28	19.23M	5.4	7
<a href="#">Canadair CRJ</a>	0.26	8.11M	2.1	3
<a href="#">Boeing MD80/MD90**</a>	0.25	40.00M	9.96	16
<a href="#">Fokker 70/100**</a>	0.21	8.99M	1.87	4
<a href="#">Boeing 737-300/400/500**</a>	0.19	62.96M	12.16	16
<a href="#">Saab 340**</a>	0.19	11.2M	2.1	3
<a href="#">Boeing 737-600/700/800/900</a>	0.13	23.53M	3.05	4
<a href="#">Airbus A320/319/321</a>	0.11	41.67M	4.63	8
<a href="#">Airbus A340</a>	0	1.91M	0	0
<a href="#">Boeing 717**</a>	0	UNK	0	0
<a href="#">Boeing 777</a>	0	4.17M	0	0

## Homework

- Please show all work
- Please refer to class handouts available on syllabus website

### Question 1:

Consider 2 airlines, A and B. Each airline has 100,000 flights during the year. The total flight hours for Airline A are 150,000 hours. The total flight hours for Airline B are 200,000 hours.

Airline A has 3 fatal crashes with the following characteristics:

- Flight with 100 passengers, 2 passengers are killed
- Flight with 8 passengers, all 8 are killed
- Flight with 50 passengers, 5 passengers are killed

Airline B has 1 fatal crash with the following characteristics:

- Flight with 200 passengers, all 200 are killed

a) Compute the following metrics for both airlines.

<b>Metric</b>	<b>Airline A</b>	<b>Airline B</b>
a) Fatal accidents per flight hour		
b) Fatal accidents per departure		
c) Passenger fatalities per departure		
d) Probability of being killed on a random flight (Barnett metric)		

b) Comment on the safety records of the 2 airlines. Would you not fly on either airline? Why?

Solution:

Question 2:

Which of the following variables would be appropriate to model with a Poisson distribution? Explain what property(s) of Poisson distributions apply.

1. Number of fatal crashes in a year
2. Number of passenger fatalities in a year

Solution:

Question 3:

Suppose there are 6,000,000 scheduled Part 121 airline flights per year. Suppose the probability of a fatal accident (per departure) is about  $10^{-7}$ . What is the probability that there are 2 or more fatal accidents per year from this flight group?

**Solution:**

$$X = \sum_{i=1}^{6,000,000} X_i$$

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 1) - P(X = 2) = 1 - e^{-A} - Ae^{-A}$$

Question 4:

A startup airline has 2 fatal crashes in its first 1,000,000 flights. Thus, the estimated rate of fatal accidents per departure is  $2 \times 10^{-6}$ . Using a 95% Poisson confidence interval, determine whether or not you can conclude that the start-up airline is less safe than the industry average of  $4 \times 10^{-7}$  fatal accidents per departure.

Repeat the calculation using a 90% confidence interval.

Note 1: Use the CHINV function in Excel.

Note 2:  $\alpha = .05$  for 95% Confidence Interval.  $\alpha = .1$  for 90% Confidence Interval.

**Solution:**

The number of fatal crashes by the start-up airline is approximately a Poisson random variable.

- The observed value is 2.

For a 95% confidence interval, set  $\alpha = .05$ .

- The lower and upper bounds for the true mean of this Poisson random variable are ...

At 95% confidence you ...

For a 90% confidence interval the lower bound

At 90% confidence

## NOTES:

### Probability of Accidents and the Poisson Distribution

The **Poisson distribution** is a discrete probability distribution. It expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate, and are **independent** of the time since the last event.

Examples of probability of a number of events:

- a) The number of soldiers killed by horse-kicks each year in each corps in the Prussian cavalry
- b) the number of V2 rocket attacks per area in England
- c) the number of light bulbs that burn out in a certain amount of time.

The random variables  $N$  that count, a number of discrete occurrences that take place during a time-interval of given length. The probability that there are exactly  $k$  occurrences ( $k$  being a non-negative integer,  $k = 0, 1, 2, \dots$ ) is

$$f(k; \lambda) = \frac{e^{-\lambda} \lambda^k}{k!},$$

where

- $e$  is the base of the natural logarithm ( $e = 2.71828\dots$ ),
- $k$  is the number of occurrences in a given time period
- $k!$  is the factorial of  $k$ ,
- $\lambda$  is a positive real number, equal to the expected number of occurrences that occur during the given interval. For instance, if the events occur on average every 4 minutes, and you are interested in the number of events occurring in a 10 minute interval, you would use as model a Poisson distribution with  $\lambda=10/4=2.5$ .

The Poisson distribution can be derived as a limiting case of the binomial distribution.

$$P(X=m) = (1-\lambda/n)^n = e^{-\lambda}$$

Example application of Poisson Distribution:

**Problem:** An insurance company has five hundred automobile insurance policies. Assume that in a given year, the number of fatal automobile accidents has a binomial distribution. On average, there is one policy out of the five hundred that will be involved in a fatal crash. What is the probability that there will be no fatal accidents (out of five hundred policies) in any particular year?

**Solution.** Let  $X$  be the number of fatal accidents in a year from a population of 500 auto insurance policies.

$p$  = average number of fatal accidents in year out of 500 policies =  $1/500$

$n$  = 500 policies

$\lambda$  = expected number of occurrences during a given period =  $500 (1/500) = 1$

$P(X=0) = e^{-1} = 0.368$

Using binomial distribution  $P(X=m) = (\lambda^m/m!) e^{-\lambda} \Rightarrow P(X=0) = (1-1/500)^{500} = 0.367$