

# AIRLINE ECONOMIC MODEL

## 1. REVENUE

$$\text{Revenue} = \text{LF} * \text{S} * \text{A} \quad (1)$$

Where:

LF = Load Factor

S = Seats

A = Average Airfare for an O-D pair for a time window

$$\text{A} = (\ln(\text{LF} * \text{S}) - \ln(\text{MS}))/\text{AS} \quad (2)$$

Where:

MS = Market Size parameter

AS = Airfare Sensitivity parameter (analogous to price elasticity)

Substituting (2) into (1):

$$\text{Revenue} = \text{LF} * \text{S} * [\ln(\text{LF} * \text{S}) - \ln(\text{MS})]/\text{AS} \quad (3)$$

## 2. COST

$$\text{Cost per Flight} = \text{BH} * \text{S} * \text{CSH} \quad (4)$$

where:

BH – gate to gate time, known as Block Hours (hours)

$$\text{CSH} = \text{Cost per Seat-Hour} (\$/\text{seat-hour}) = [ \text{NFR} + (\text{FBR} * \text{FP}) ] \quad (5)$$

Where:

NFR – non-fuel rate (\$/seat-hour)

FBR – fuel burn per seat-hour (gallons/seat-hour)

FP – fuel price (\$/gallon)

Substituting (5) into (4)

$$\text{Cost per Flight} = \text{BH} * \text{S} * [ \text{NFR} + (\text{FBR} * \text{FP}) ] \quad (6)$$

NFR can be approximated by linear function:

$$\text{NFR} = (\text{NFR\_Slope} * \text{Seats}) + \text{NFR\_YIntc} \quad (7)$$

FBR can be approximated by linear function:

$$\text{FBR} = (\text{FBR\_Slope} * \text{Seats}) + \text{FBR\_YIntc} \quad (8)$$

Substituting (7) and (8) into (6)

$$\text{Cost per Flight} = \text{BH} * \text{S} * [ \{(\text{NFR\_Slope} * \text{S}) + \text{NFR\_YIntc}\} + \{(\text{FBR\_Slope} * \text{S}) + \text{FBR\_YIntc}\} * \text{FP} ]$$

### 3. PROFIT

$$\text{Profit} = \text{Revenue} - \text{Cost per Flight} \quad (9)$$

Substituting (7) and (8) into (9)

$$\text{Profit} = [ \text{LF} * \text{S} * [ \ln(\text{LF} * \text{S}) - \ln(\text{MS}) ] / \text{AS} ] - [ \text{BH} * \text{S} * [ \{(\text{NFR\_Slope} * \text{S}) + \text{NFR\_YIntc}\} + \{(\text{FBR\_Slope} * \text{S}) + \text{FBR\_YIntc}\} * \text{FP} ] ] \quad (10)$$

$$= [ \text{LF} * \text{S} * [ \ln(\text{LF} * \text{S}) - \ln(\text{MS}) ] / \text{AS} ] -$$

$$[ \text{BH} * \text{S}^2 * \text{NFR\_Slope} + \text{BH} * \text{S} * \text{NFR\_YIntc} + \text{BH} * \text{S}^2 * \text{FBR\_Slope} * \text{FP} + \text{BH} * \text{S} * \text{FP} * \text{FBR\_YIntc}$$

$$\text{S}^2 * \text{BH} (\text{NFR\_Slope} + \text{FBR\_Slope}) + \text{S} * \text{BH} * (\text{NFR\_YIntc} + (\text{FP} * \text{FBR\_YIntc})) ]$$

$$= [ \text{LF} * \text{S} * [ \ln(\text{LF} * \text{S}) - \ln(\text{MS}) ] / \text{AS} ] -$$

$$[ \text{BH} * \text{S}^2 * (\text{NFR\_Slope} + (\text{FBR\_Slope} * \text{FP})) + \text{BH} * \text{S} * (\text{NFR\_YIntc} + (\text{FP} * \text{FBR\_YIntc})) ]$$

$$= [ \text{LF} / \text{AS} * [ \ln(\text{LF} * \text{S}) - \ln(\text{MS}) ] ] -$$

$$[ \text{BH} * \text{S}^2 * (\text{NFR\_Slope} + (\text{FBR\_Slope} * \text{FP})) + \text{BH} * \text{S} * (\text{NFR\_YIntc} + (\text{FP} * \text{FBR\_YIntc})) ]$$

#### 4. PROFIT MAXIMIZATION

To find the Aircraft Seat Size for maximum profit, take the derivative of Profit equations (use chain rule for  $S \ln(LF * S)$ ), set equal to zero, and solve for S.

Using chain rule

$$dP/dS = \left[ LF/AS \left[ \ln(LF*S) \frac{d(S)}{dS} + S \frac{d(\ln(LF*S))}{dS} - \ln(MS) \frac{d(S)}{ds} \right] - \left[ (2*S* BH (NFR\_Slope + (FBR\_Slope * FP)) + BH * (NFR\_YIntc + (FP * FBR\_YIntc)) \right] \right]$$

$$dP/dS = \left[ LF/AS \left[ \ln(LF*S) + (LF*S)*(1/(LF*S)) - \ln(MS) \right] - \left[ (2*S* BH (NFR\_Slope + (FBR\_Slope * FP)) + BH * (NFR\_YIntc + (FP * FBR\_YIntc)) \right] \right]$$

$$dP/dS = \left[ LF/AS (\ln(LF * S) + 1 - \ln (MS)) \right] - \left[ (2*S* BH (NFR\_Slope + (FBR\_Slope * FP)) + BH * (NFR\_YIntc + (FP * FBR\_YIntc)) \right]$$

To solve for S, substitute a 2<sup>nd</sup> order polynomial to substitute for  $\ln(LF * S)$ , set derivative to zero and solve for S.

Linearize  $\ln(LF * S)$  for range  $LF * S = [50, 200]$ ,  $\ln(LF * S) = 0.0092(LF*S) + 3.4621$  ( $R^2 = 0.995$ )

Substitute a 2<sup>nd</sup> order polynomial to substitute for  $\ln(LF * S)$

$$dP/dS = \left[ LF/AS(1 - \ln (MS)) + LF/AS(-4E-05(LF*S)^2 + 0.0183(LF*S) + 3.1459) \right] - \left[ (2S* BH (NFR\_Slope + (FP*FBR\_Slope)) + BH * (NFR\_YIntc + (FP * FBR\_YIntc)) \right]$$

$$0 = \left[ LF/AS(1 - \ln (MS)) + LF/AS(-2E-05(LF*S)^2 + 0.0143(LF*S) + 3.3538) \right] - \left[ (2S* BH (NFR\_Slope + (FP*FBR\_Slope)) + BH * (NFR\_YIntc + (FP * FBR\_YIntc)) \right]$$

$$0 = -2E-05 (LF^3/AS) S^2 + S \left[ (0.0143 LF^2/AS) - 2BH (NFR\_Slope + (FP*FBR\_Slope)) \right] + 3.3538 (LF/AS) + LF/AS (1-\ln(MS)) - BH (NFR\_YIntc + (FP * FBR\_YIntc))$$

Using Quadratic Method, the Roots of the 2<sup>nd</sup> order polynomial are

$$(-b \pm \sqrt{b^2 - 4ac}) / 2a$$

Where:

$$a = -2E-05 (LF^3/AS)$$

$$b = [ (0.0143 LF^2/AS) - 2BH (NFR\_Slope + (FP * FBR\_Slope)) ]$$

$$c = 3.3538 (LF/AS) + LF/AS (1 - \ln(MS)) - BH (NFR\_YIntc + (FP * FBR\_YIntc))$$

## 5. EXAMPLE: PROFIT MAXIMIZATION

The complex relationship between revenue, costs and profit is illustrated in Figure 4. The relationship exhibits four “economic operating points.” The “maximum profit” point identifies the combination of airfares/demand and costs that generate the maximum profit (i.e. marginal profit equals zero). The “maximum revenue” point identifies the combination of airfares and demand that generate the most revenue (i.e. marginal revenue equals zero). Due to the exponential shape of the airfare elasticity and the slope of the cost of transportation function, the maximum profit point will always transport fewer passengers than the maximum revenue point. For market pairs in which competition exists in the demand window, the operating point selected by airlines will lie between the maximum profit and maximum revenue point.

The “zero profit” point identifies the number of passengers that would be transported if the airline was willing to operate without profit. The “zero revenue” point is a theoretical measure of the total number of passengers that would avail themselves of the transport service if it were available at zero cost. This operating point is used as a reference measure for affordability metrics.

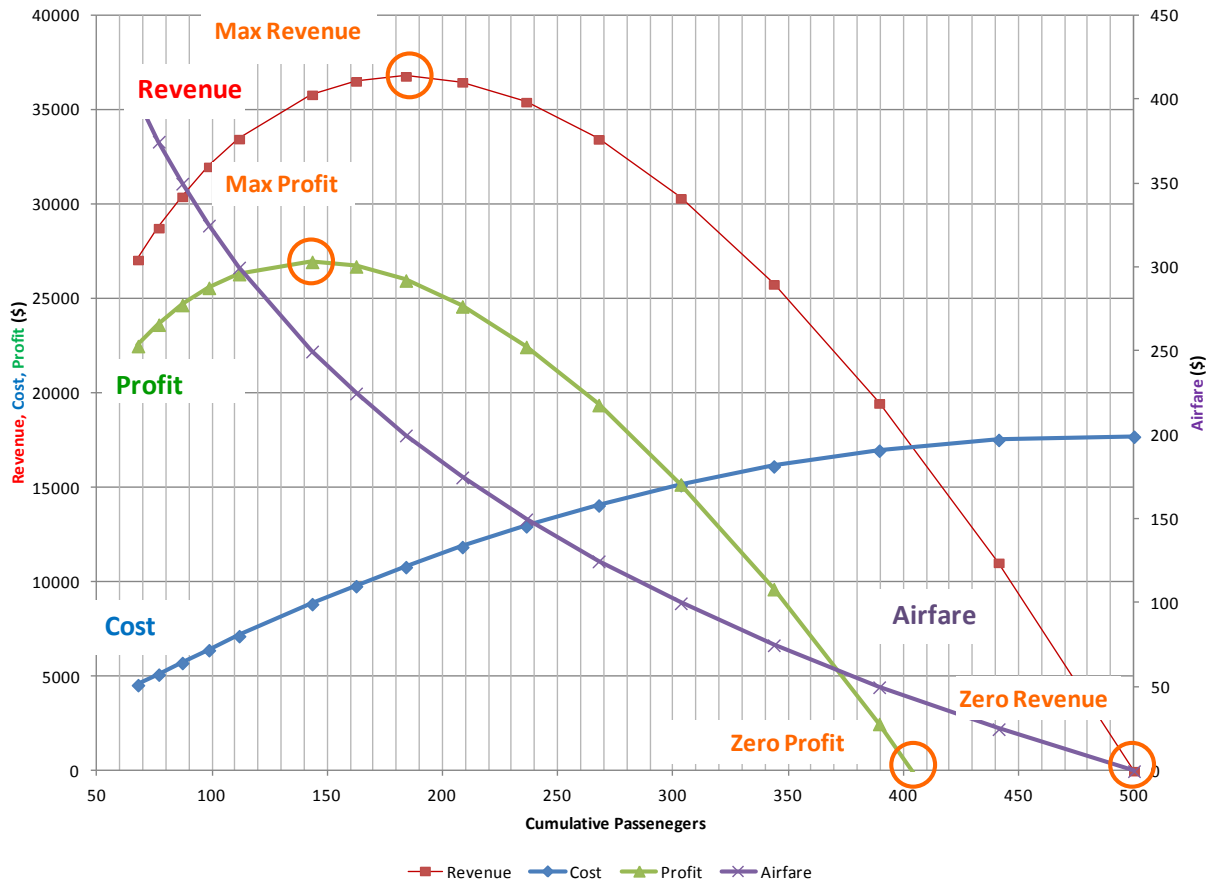


Figure 4: Complex non-linear relationship between revenue (red), based on airfare vs demand (magenta), cost (blue), and profit (green). To maximize profit, airlines provide the seat capacity at the maximum profit operating point. The slope of the cost curve determines the maximum profit point which establishes the airfare that determines the affordability of travel.

### The Effect of Increasing Fuel Price

An increase in fuel price from \$1/gallon to \$4/gallon results in an increase in flight operational costs (Figure 5). This shifts the cost curve up (blue dashed line). As result, the profit curve is shifted down and to the left. The next effect of the increase in fuel price in the presence of a profit maximizing scheme is to transport **fewer passengers paying higher airfares**.

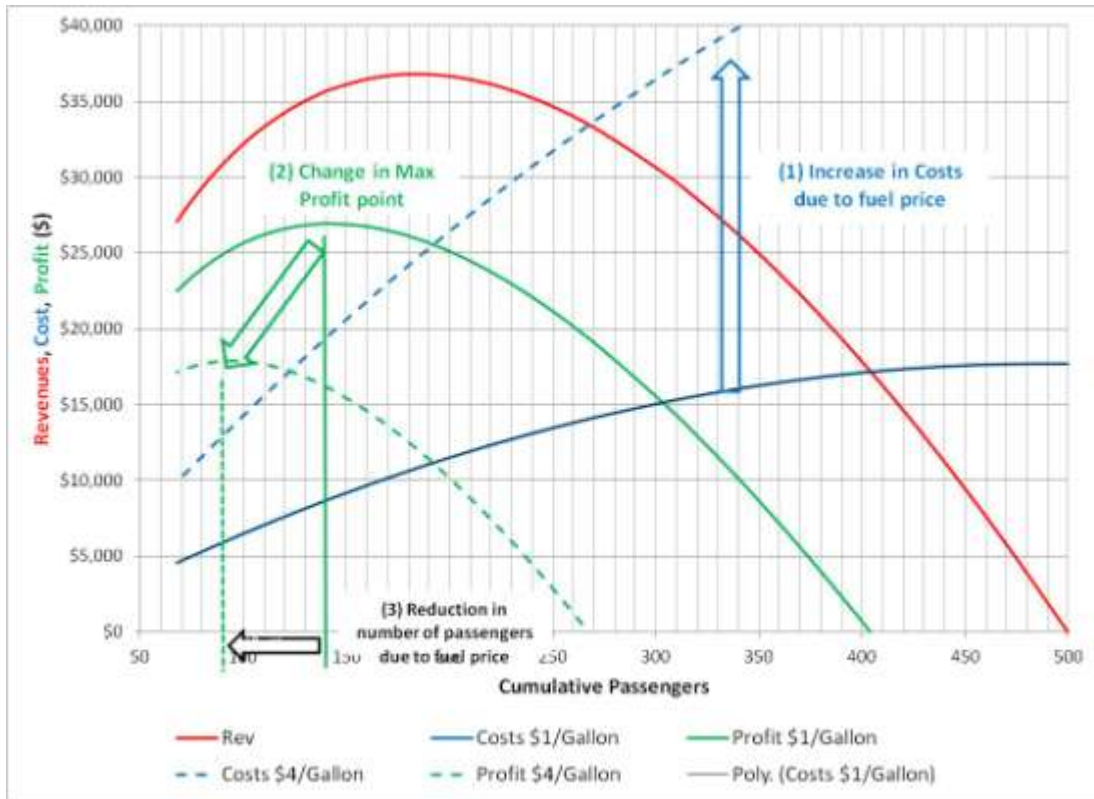


Figure 5: An increase in fuel price from \$1/gallon to \$4/gallon results in an increase in flight operational costs (Figure 5). This shifts the cost curve up (blue dashed line). As result, the profit curve is shifted down and to the left. The next effect of the increase in fuel price in the presence of a profit maximizing scheme is to transport ***fewer passengers paying higher airfares.***

## 6 CASE STUDY

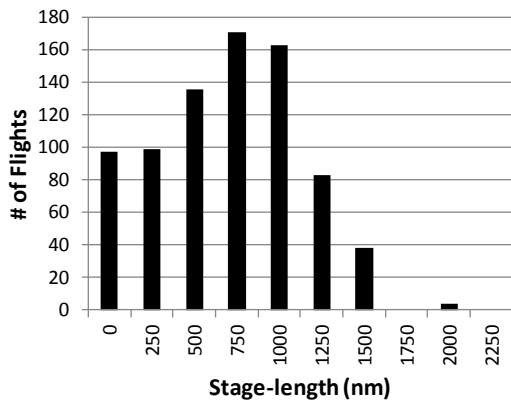
The impact of increasing fuel prices across an airline network is illustrated for a case study for a mid-west hub-and-spoke network located in central Texas (i.e. Dallas/Forth Worth). The network is composed of 102 markets that have historically served DFW with average stage-length of 850nm (standard deviation 427 nm). The distribution of stage-lengths by flights is shown in Figure 7-a.

The revenue functions for were derived from Market Size and Airfare Sensitivity data for markets historically served by DFW from BTS data from 2005 to 2011. The aircraft cost data is derived from BTS data from 2005-2011 and represents all aircraft in the U.S. domestic fleet during that period. The model was calibrated to maximize airline profit by meeting the demand at \$1/gallon fuel price using single aisle aircraft configured for 120 seats. The ASMs, Total Revenue, Average Airfare and Average Aircraft Size of the model are analyzed for fuel price increasing from \$1/gallon to \$5/gallon (see Figures 7-b, c, d).

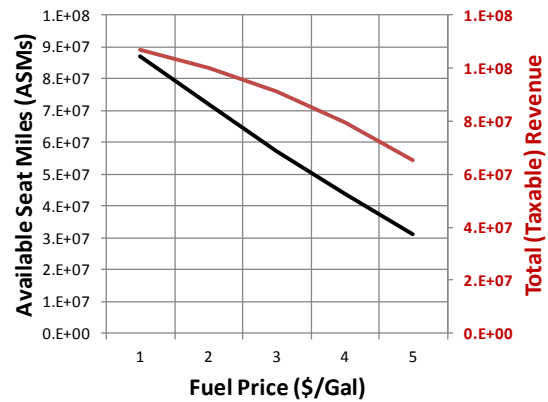
Results of the model indicate a reduction of 14 million ASMs for every \$1/gal increase in fuel price. The reductions in ASMs occur due to a 20 seat decrease in average aircraft size with each \$1/gal increase in fuel price. This is accompanied by a \$23 increase in average airfare for every \$1/gal increase in fuel price. The combination of fewer passengers paying higher airfares, however results in a reduction in taxable revenue. Taxable revenue does not decay as fast as ASMs - \$10.5M for every \$1/gal increase in fuel price.

The variance in Average Airfare and Average Aircraft Size represents the distribution over all the markets serviced by the hub. The increasing variance in average airfare as fuel prices increase, along with the decreasing variance in aircraft size is indicative of a shift in economic operating point to capture the inelastic higher paying passengers on the major “trunk line” routes.

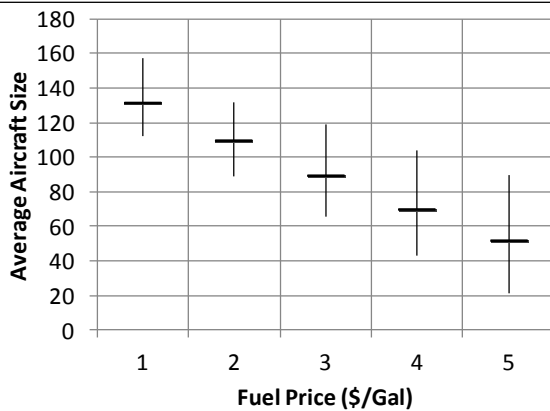
To maintain the airfares and aircraft size for fuel prices at \$1/gallon, when fuel prices increase to \$4/gallon, the model shows that aircraft operational costs must be decreased by 47%.



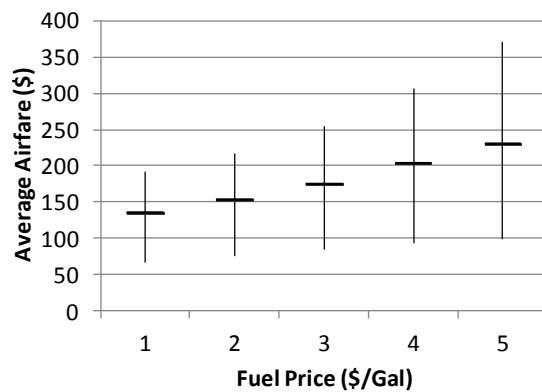
(a)



(b)



(c)



(d)

**Figure 7: Results of a case study for the 102 markets with service to/from DFW (stage-length  $\mu = 850\text{nm}$ ,  $\sigma = 427\text{nm}$ ). Increasing fuel prices result in a reduction in Available Seat Miles(ASMs) (b) due to a decrease in aircraft size by 20 seats per \$1/gallon increase in fuel price (c). This is accompanied by an increase of \$23 in airfare per \$1/gallon increase in fuel price (d).**