Capacity of a Single Runway

Learning Objectives:

The student will understand and know how to use the following terms

1. Maximum Throughput Capacity (MCT)
2. Runway Occupancy Time (ROT)
3. Simultaneous Runway Occupancy (SRO)
4. Arrival Wake Vortex Separation Distance
5. Inter-arrival Separation Distance (S_{ij})
6. Inter-arrival Separation Time (T_{ij})
7. Lead-Follow Pair Compression
8. Lead-Follow Pair Separation Distance (s_{ij})
9. Fleet Mix and Lead-Follow Pair Probabilities (p_{ij})

The student will be able to create and use a Maximum Throughput Capacity model for a single runway.

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Maximum Throughput Capacity (MCT) is the fundamental measure of the capacity of the runway. MCT defines the average number of movements (arrivals and/or departures) that can be performed on the runway in one hour.

MCT makes the following assumptions:

(i) there is a continuous supply of arrivals and/or departures
(ii) Air Traffic Control rule - no Simultaneous Runway Occupancy (SRO)
(iii) Air Traffic Control rule - safe Wake Vortex Separation Distances between two flights
(iv) Static fleet mix (i.e. type of aircraft do not change)
(v) Approach procedure does not change

As a consequence, MCT is a theoretic measure of runway capacity and is represented as an average capacity for the runway.

The following sections describe a method for estimating Maximum Throughput Capacity (MCT) for a runway. The method is shown for arriving aircraft, but can be modified to cover departures only and mixed arrivals and departures.

There are five considerations in determining the MCT of a runway.

1. ATC Safety Rule: no Simultaneous Runway Occupancy (SRO)
2. ATC Safety Rule: Maintain Wake Vortex Separation Distance between lead and follow aircraft
3. ATC Controller/Pilot Separation Control Accuracy: ATC/Pilots insert a buffer distance to void violating separation rules
4. Fleet Mix: determines the type of aircraft in the lead-follow pairs. The type of aircraft determines the separation distance used. Small aircraft following large aircraft require longer distances than large aircraft following large aircraft.
5. Final Approach Path Distance: the length of time lead-follow aircraft fly the approach in pairs determines the additional buffer spacing required for compression (faster aircraft following a slower aircraft) and separation (slow aircraft following a fast aircraft).

The table below includes the equations used to model MCT. The rows in the table show the equations for MCT for the no Simultaneous Runway Occupancy rule, the Wake Vortex Separation Rule, and the Controller/Pilot Separation Buffer. The 3rd column includes the equation for a homogeneous fleet mix (i.e. all the same aircraft). The 4th column includes the equations for a non-homogeneous fleet mix given probabilities for each type of aircraft and the probabilities for a lead-follow pair.

	Parameter	**MCT for Homogeneous Fleet Mix**	**MCT for Non-Homogeneous Fleet Mix**
No Simultaneous Runway Occupancy (SRO)	Runway Occupancy Time (ROT)	MCT = 3600/ROT	MCT = 3600/E[ROT] $E[ROT] = \Sigma_i\Sigma_j (p_{ij} *ROT_i)$
Wake Vortex Separation Distance	Inter-arrival Separation Distances	$MCT = 3600/(s_{ij}/v_j)$	$MCT = 3600/E[T_{ij}]$ $E[T_{ij}] = \Sigma_i\Sigma_j (p_{ij} *(T_{ij}))$ $T_{ij} = s_{ij}/v_j$ for compression case $T_{ij} = (r + s_{ij}/v_j) – r/ v_i$ for separating case
ATC Accuracy of Separation Control	ATC Buffer Separation Distance (10 to 25 secs)	$MCT = 3600/((s_{ij}/v_j)+b)$	$MCT = 3600/E[T_{ij}]$ $E[T_{ij}] = \Sigma_i\Sigma_j (p_{ij} *((T_{ij})+b))$

Maximum Throughput Capacity for Homogeneous Fleet Mix

The MCT for a runway, to meet only the Simultaneous Runway Occupancy (SRO) rule is determined by the Runway Occupancy Time (ROT) of the aircraft.

MCT = 3600 seconds/ROT

The average ROT typically ranges from 45 seconds for small aircraft to 70 seconds for large aircraft. The MCT for a fleet of homogeneous aircraft with ROT of 60 seconds:

MCT = 3600 seconds/ROT = 3600/60 = 60 flights per hour

The MCT for a runway to account for the Wake Vortex Separation Distance requirements is determined by the separation distance.

Wake vorticies are generated off the wing-tips of an aircraft. Air flowing over the wing splits into two streams. The air flowing over the curved upper surface of the wing increases it's velocity, reducing the density of the air (compared to the air flowing under the wing) and generates the lift force that enables the airplane to fly. The greater the weight of the aircraft, the greater the lift force that must be generated.

Air also spills over the edge of the wings creating two tubes of circulating air that trail behind the aircraft. The circulating air is known as a Wake Vortex and has sufficient strength to cause an aircraft following to closely to roll (i.e. raise one wing and dip the other). The sudden rolling motion can cause an aircraft to lose lift and go into a dive. When the aircraft is to close to the ground (e.g. departure or approach), there may not be sufficient altitude to correct the dive. In other cases, the sudden rolling motion can cause the aircraft structure to break. This is what happened in the American Airlines Flight 587 accident.

When the Wake Vorticies leave the wing, air friction and air turbulence (e.g wind) disrupt the vorticies and cause the wakes to dissipate. The direction of the circulation also causes the wake to sink. A typical wake will sink approximately 800 feet from the point it was generated. A typical wake will exist at the point it was generated for about 90 seconds.

Learn More About It:

On November 12, 2001, about 09:16 eastern standard time, American Airlines flight 587, an Airbus Industrie A300-605R, N14053, crashed into Belle Harbor, a New York City residential area, shortly after takeoff from John F. Kennedy International Airport.

The plane's vertical stabilizer and rudder separated in flight and fell into Jamaica Bay, about 1 mile north of the main wreckage site. The plane's engines subsequently separated in flight and fell several blocks north and east of the main wreckage site. All 260 people aboard the plane and 5 people on the ground died, and the impact forces and a post-crash fire destroyed the plane.

NTSB Accident Report: In-Flight Separation of Vertical Stabilizer American Airlines Flight 587 Airbus Industrie A300-605R, N14053 Belle Harbor, New York November 12, 2001: Aircraft **Accident Report**. NTSB/AAR-04/04
http://www.ntsb.gov/publictn/2004/AAR0404.pd

NTSB Video: http://www.ntsb.gov/events/2001/aa587/flight_path_web01.wmv

History Channel video on Flight 587
http://www.youtube.com/watch?v=5beNH9EKnps

The circulation strength and duration of the wake vortex is a function of the weight of the aircraft. Heavier aircraft will generate stronger and longer lasting wake vorticies than lighter aircraft.

To ensure the safety of following aircraft, AirTraffic Controllers are required to separate lead-follow aircraft by a Minimum Separation Distance as described in the table below: The larger, the lead aircraft and the smaller the follow aircraft, the larger the distance. The separation distance is the distance required as the lead aircraft crosses the runway threshold.

		Trailing aircraft			
leading		1(H)	2(L)	3(M)	4(S)
aircraft	I(H)	4	5	5	6
	2(L)	2.5	2.5	2.5	4
S =	3(M)	2.5	2.5	2.5	4
	4(S)	2.5	2.5	2.5	2.5

The MCT for a runway, to meet only the Wake Vortex Separation Rule, for a homogeneous fleet mix, is determined by the time between arriving aircraft at the runway threshold, known as inter-arrival time (t). The inter-arrival time between two aircraft is computed based on the distance between the aircraft (s) at the runway threshold and the groundspeed of the aircraft (v).

MCT = 3600/(s/v),   where t = s/ v

For a Heavy following a Heavy, the inter-arrival time is (4nm/150 knots)*3600 secs/1hr = 96 seconds

MCT = 3600 seconds/Inter-arrival time = 3600/96 = 37.5 flights per hour

Note that for Heavy aircraft, the MCT of 37.5 flights per hour due to Wake Vortex Separation distance is more restrictive than the 60 flights per hour due to Runway Occupancy Time.

The MCT for a runway with homogeneous fleet mix to account for the ATC/Controller Separation Buffer for maintaining Wake Vortex Separation Distance requirements is determined by the separation distance plus the buffer distance.

The separation distance between the lead and follow aircraft is established by collaboration between the ATC controller and the pilot. The controller, monitoring radar, provides instructions to the lead and/or follow aircraft to speed-up or slow down to achieve and maintain the separation. The pilots, monitoring the aircraft in front of them, will follow the controller instructions by adjusting thrust, airbrakes or rate of descent.

The natural tendency of the controllers and pilots is to be conservative and err on the side of caution creating larger separation distances. In the United States the average separation distances exceed the minimum separation distance by a buffer (b) of 10 to 25 seconds.

$$MCT = 3600/((s/v) + b), \qquad \text{where } t = (s/v) + b$$

For a Heavy following a Heavy, the inter-arrival time is (4nm/150 knots)*3600 secs/1hr = 96 seconds. An additional 10 seconds of buffer yields an inter-arrival time of 106 seconds.

MCT = 3600 seconds/Inter-arrival time = 3600/(96 secs + 10 secs) = 34 flights per hour

Maximum Capacity Throughput (MCT) for a homogeneous fleet mix is the minimum capacity throughput for the SRO, Wake Vortex, and ATC/Controller Buffer values:

$$MCT = \text{Min}\,(MCT_{SRO}, MCT_{WVSD}, MCT_{WVSDB})$$

Since the ATC/Controller Buffer increases the inter-arrival time between flights, the equation for MCT can be simplified to:

$$MCT = \text{Min}\,(MCT_{SRO}, MCT_{WVSDB})$$

Maximum Throughput Capacity for a Non-Homogeneous Fleet Mix

The next step is add the effect of fleet mix into the equations.

Fleet mix is represented by the probabilities of a each type of aircraft by the following table.

Fleet Mix Probability Vector **P**

Fleet Mix (a/c type)	Probability of Type of Aircraft (prob.)
I(H)	0.2
2(L)	0.35
3(M)	0.35
4(S)	0.1

The MCT for a runway, to meet only the Simultaneous Runway Occupancy (SRO) rule, for a non-homogeneous fleet mix, is determined by: (i) the Runway Occupancy Time (ROT) of the aircraft, and (ii) the probability of the lead-follow (of Fleet Mix).

The average ROT of a fleet mix is shown below:

Runway Occupancy Time (ROT) for Fleet Mix

Fleet Mix (a/c type)	Runway Occupancy Time (sec)
1(H)	70
2(L)	60
3(M)	55
4(S)	50

The MCT for a runway to meet only the SRO rule is defined as follows:

MCT = 3600/E[ROT]

Where E[ROT] is defined as the expected value, or average Runway Occupancy Time given the fleet mix for this runway. This is computed by summing the probability of Lead aircraft (i), $p_{i,}$, times the ROT for the lead aircraft (i).

$$E[ROT] = \Sigma_i (p_i * ROT_i)$$

For the example data provided above,

E[ROT] = (70 * 0.2) + (60 *0.35) + (55 * 0.35) + (50 * 0.1) = 59.25 seconds

MCT = 3600/59.25 = 60.7.

The MCT for a runway, to meet only the Wake Vortex Separation Rule, for a non-homogeneous fleet mix, is determined by; (i) the separation distance between the lead and the follow (s_{ij}), (ii) the groundspeed of the aircraft (v_j), and (iii) the probability of a lead-follow pair (p_{ij}).

When the lead aircraft crosses the runway threshold, the follow aircraft will be the required distance behind. The time between the lead and follow aircraft is known as the inter-arrival time (t_{ij}) and is represented by the Inter-arrival Time Matrix **T**.

T = Inter-arrival Time Matrix

		Trailing aircraft			
leading		1(H)	2(L)	3(M)	4(S)
aircraft	I(H)	96	157	207	320
	2(L)	60	69	107	222
T =	3(M)	60	69	82	196
	4(S)	60	69	82	100

Each cell (t_{ij}) in the Inter-Arrival Time Matrix **T**, is computed as follows:

$E[T_{ij}] = \Sigma_i\Sigma_j (p_{ij} *(T_{ij}))$

Where:

$T_{ij} = s_{ij}/v_j$ for compression case
$T_{ij} = ((r + s_{ij})/v_j) – (r/ v_i)$ for separating case

p_{ij} is the Probability of the Lead/Follow Pairs. Each cell is computed by the $p_i * p_j$. For example, for the Fleet mIx provided above **P**, the probability of a Heavy following a Heavy = 0.2 * 0.2 = 0.04

Probability of Leading/Trailing Pairs

		Trailing aircraft			
leading		1(H)	2(L)	3(M)	4(S)
aircraft	I(H)	0.04	0.07	0.07	0.02
	2(L)	0.07	0.12	0.12	0.04
P =	3(M)	0.07	0.12	0.12	0.04
	4(S)	0.02	0.04	0.04	0.01

The are two equations used to compute T_{ij}: the compression case, and the separation case.

Compression Case: Lead Slower than Follow

When the lead aircraft is slower than the follow aircraft, the follow aircraft will catch up to the lead aircraft as they fly constant speed down the approach path. To ensure that the separation distance is maintained down to the runway threshold, the ATC controller separates the follow aircraft from the lead aircraft according to the required separation distance, plus an additional compression distance. The *compression distance* is the additional distance used by the follow aircraft as it catches up to lead aircraft such that the separation distance is achieved at the runway threshold. The compression distance required is the a function of length of the approach path (r) and the relative groundspeed between the lead and follow aircraft ($V_j – V_i$) and is represented as a time as follows:

Compression Time = $r/(V_j - V_i)$

A longer approach path will provide more time for the follow to catch up to the lead, and require a greater compression time.

Compression cases: H-H, H-L, H-M, H-S, L-L, L-M, L-S, M-M, M-S, S-S.

Separation case: Lead Faster than Follow

When the lead aircraft is faster than the follow aircraft, the follow aircraft drops back from the lead aircraft as they fly constant speed down the approach path. To ensure that the separation distance is maintained down to the runway threshold, the ATC controller separates the follow aircraft from the lead aircraft according to the required separation distance at the start of the approach. The *separation distance* is the additional distance at the runway threshold caused by a lead faster than a follow.

The separation distance required is a function of length of the approach path (r) and the relative groundspeed between the lead and follow aircraft ($V_j - V_i$) and is represented as a time as follows:

$T_{ij} = ((r + s_{ij})/v_j) - (r/ v_i)$ for separating case

Separating Cases: S-M, S-L, SH, M-L, M-H, H-L.

For the fleet mix, Runway Occupancy Time, Ground Speed, and Approach path of 5nm, the Inter-arrival Time Matrix tij is shown below.

Inter-Arrival Time (sec)

		Trailing aircraft			
leading		1(H)	2(L)	3(M)	4(S)
aircraft	I(H)	96	157	207	320
	2(L)	60	69	107	222
T =	3(M)	60	69	82	196
	4(S)	60	69	82	100

MCT = 3600 seconds/E[t_{ij}] = 34 flights per hour

The MCT for a runway with non-homogeneous fleet mix to account for the ATC/Controller Separation Buffer for maintaining Wake Vortex Separation Distance requirements is determined by the separation distance plus the buffer distance.

The natural tendency of the controllers and pilots is to be conservative and err on the side of caution creating larger separation distances. In the United States the average separation distances exceed the minimum separation distance by a buffer (b) of 10 to 25 seconds.

$$MCT = 3600/E[t_{ij}]$$

Where $t_{ij} = T_{ij} + b$

For the fleet mix, Runway Occupancy Time, Ground Speed, ATC/Controller Buffer, and Approach path of 5nm, the Inter-arrival Time Matrix tij is shown below.

Inter-Arrival Time (sec)

		Trailing aircraft			
leading		1(H)	2(L)	3(M)	4(S)
aircraft	I(H)	106	167	217	330
	2(L)	70	79	117	232
T =	3(M)	70	79	92	206
	4(S)	70	79	92	110

MCT = 3600 seconds/E[t_{ij}] = 31 flights per hour

Maximum Capacity Throughput (MCT) for a non-homogeneous fleet mix is the minimum capacity throughput for the SRO, Wake Vortex, and ATC/Controller Buffer values. Since the ATC/Controller Buffer increases the inter-arrival time between flights, the equation for MCT can be simplified to:

$$MCT = \text{Min}\,(MCT_{SRO}, MCT_{WVSDB})$$