The Standard Atmosphere

SYST 460/560
Intro to Air Transportation System Engineering
Instructor: Lance Sherry (Ph.D.)

P, T, D, a
Motivation

• Aerodynamic and propulsive forces acting on aircraft depend on:
  – Local pressure (P)
  – Local temperature (T)
  – Local density (D)
  – Sonic velocity (a)

• How do P, T, D, a change as a function of altitude
Learning Objectives

Knowledge
• Hydrostatic equation
• Equation of state (for air as a perfect gas)
• Lapse Rate Equation
• Troposphere
• Stratosphere
• Sonic Velocity
• Standard Atmosphere

Skills
• Derive equations for (T, P, D, a) from basic equations for Troposphere and Stratosphere
• Calculate Ratios for Standard Atmosphere
• Calculate T, P, D, a for Standard Atmosphere
Composition of Atmosphere

• Air is treated as a perfect dry gas
  – 78% Nitrogen
  – 22% Oxygen
  – traces of other gases like Hydrogen, Carbon dioxide, ...
Properties of Atmosphere

• Density = $f(\text{Altitude})$
  – Density decreases as altitude increases

• Pressure = $f(\text{Altitude})$
  – Pressure decreases as altitude increases

• Temperature = $f(\text{Altitude})$
  – Temperature decreases as altitude increases
  – drops 1 deg C for every 1000 ft increase in altitude
Troposphere/Stratosphere

Altitude = Height above ground

36,089 feet

Density so low, no change in T

Stratosphere (turbulent)

Troposphere

Density enough, so change in T
Standard Atmosphere

• Standard Atmosphere defines values for (P, T, D, a) as a function of altitude

• Assumptions:
  – (1) atmosphere is static,
  – (2) rotates with the Earth

• Subsonic aircraft: surface to 45,000ft
Basic Equation #1 – Hydrostatic Equation

• The difference in pressure \((dp)\) between two altitudes \((dH)\), is equal to the weight (mass * gravitational constant).

• Relates pressure \((p)\) and density \((\rho)\) to height

\[
dp = - \rho g \, dH
\]

– \(p\) = barometric pressure \((lb/ft^2)\)
– \(\rho\) = density \((sl/ft^3)\)
– \(g\) = gravitational constant \((ft/sec^2)\)
– \(H\) = Height in Standard Atmosphere \((ft)\)
Basic Equation #2 - Equation of State for Air (as a Perfect Gas)

• $P = \rho \times R \times T$

• Relates the pressure ($p$) and density ($\rho$) to the Temperature ($T$)
  – $R = \text{gas constant for air} = 287.053 \text{ joules/kg-deg K} = 1716.551 \text{ ft-lb/sl-degR}$
  – $P_0 = \text{standard sea-level pressure} 101325 \text{ n/m}^2 = 2116.22 \text{ lb/ft}^2 = 29.9213 \text{ in Hg at } T_0 = 518.67 \text{ deg R}$
  – $P_0 = \text{standard sea-level density} 1,22500 \text{ kg/m}^3 = 0.00237691 \text{ sl/ft}^3$
Basic Equation #3 – Temperature vs Altitude

• **Troposphere** (surface (-1000 ft) to \( H_T = 36,089 \) ft)
  
  \[- T_T = T_0 + (L \cdot H) \quad 0 \leq H \leq H_T \text{ changes with altitude} \]

• **Stratosphere** (greater than \( H_T = 36,089 \) ft)
  
  - Isothermal layer (i.e. constant temperature)
  
  \[- T_S = T_0 + (L \cdot H_T) \quad H > H_T \text{ does not change with altitude} \]

• \( L = \frac{dT}{dH} = \text{thermal lapse rate} = -6.5 \text{ deg K per km} = 5.5^\circ F/1000 \text{ feet} = \)

• \( T_0 = \text{standard sea level temperature} = 288.15 \text{ deg K} = 518.67 \text{ deg R} = 15 \text{ deg C} = 59 \text{ deg F} \)
Basic Equation #3 – Temperature vs Altitude

• Below 36,089 feet,
  – Ambient Temperature (°R) = -3.566° * (Altitude/1000)

• Above 36,089 feet
  – Ambient Temperature (°R) = 389.988°
Basic Equation #4 – Sonic Velocity

• Sonic Velocity = \( a \)
  \[
a = \sqrt{\gamma \times R \times T}
\]
  – \( \gamma \) = ratio of specific heats for air = 1.4 (dimensionless)
  – \( a_0 \) = standard sea-level velocity 340.294 m/sec = 1116.45 ft/sec
Normalized Equations for Troposphere

- **Normalized Temperature Ratio**
  \[ \Theta = \frac{T}{T_0} = 1 + L \left( \frac{H}{T_0} \right) \]

- **Normalized Pressure Ratio**
  \[ \delta = \frac{P}{P_0} = \left[ 1 + \frac{H}{(T_0/L)} \right] \]

- **Normalized Density Ratio**
  \[ \sigma = \delta / \Theta = \left[ 1 + H(T_0/L) \right]^{-(1+g/LR)} \]

- **Normalized Sonic Velocity**
  \[ \mu = \frac{a}{a_0} = \sqrt{\Theta} \]
Normalized Equations for Stratosphere

• Normalized Temperature Ratio
\[ \Theta_S = \frac{T_s}{T_0} = 1 + H_T \left( \frac{T_0}{L} \right) \]

• Normalized Pressure Ratio
\[ \delta_S = \delta_T \exp \left[ - \frac{(H - HT)}{(RT_S/g)} \right] \]

• Normalized Density Ratio
\[ \sigma_S = \frac{\delta_S}{\Theta_S} = \frac{\delta_T}{\Theta_S} \{ \exp \left[ - \frac{(H - HT)}{(RT_S/g)} \right] \} \]

• Normalized Sonic Velocity
\[ \mu_S = \frac{a}{a_0} = \sqrt{\Theta_S} \]
Numerical Constants

$T_0/L = -145,442 \text{ ft}$

$-g/LR = 5.255913 \text{ (dimensionless)}$

$T_S = 389.97 \text{ deg R}$

$\delta_T = 0.223359 \text{ (dimensionless)}$

$RT_S/g = 20,805.7\text{ ft}$
**Numerical Equations**

**Troposphere**

\[ \theta = 1 - (6.8753 \times 10^{-6})^5 \]

Pressure Altitude

\[ \delta = 1 - (6.88 \times 10^{-6})^5 \]

Pressure Altitude

\[ \sigma = \frac{\delta}{\theta} \]

\[ \mu = \frac{a}{a_0} = \sqrt{\theta_s} \]

**Stratosphere**

\[ \theta = \theta_s = \frac{T_s}{T_0} = 0.751865 \]

\[ \delta = 0.22336 \exp\left(\frac{36,089 - \text{Pressure Altitude}}{20,805.7}\right) \]

\[ \sigma = \frac{\delta}{\theta} \]

\[ \mu = \frac{a}{a_0} = \sqrt{\theta_s} = 0.867107 \]
# Tabulated Values

<table>
<thead>
<tr>
<th>H (ft)</th>
<th>( T_0 ) (deg R)</th>
<th>( P_0 ) (lb/ft²)</th>
<th>( P_0 ) (sl/ft³)</th>
<th>( a_0 ) (ft/sec)</th>
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Graphed Values

Altitude or H (ft)

θ  δ  s  μ
<table>
<thead>
<tr>
<th>Altitude (ft)</th>
<th>Pressure (in. Hg)</th>
<th>Temperature (F.)</th>
<th>Density, slugs per cubic foot</th>
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</table>
**Test Yourself**

**Q:** Compute the Standard Atmosphere Temperature at H=35,000 ft

**A:**

1. \[ \theta = \frac{T}{T_0} \]
2. \[ T = \theta \times T_0 = 0.759354 \times 518.67 \text{ (deg R)} \]
3. \[ T = 3886.9 \text{ (deg R)} \]
4. \[ T = 1 \text{ (degR)}/-272.594444\text{(deg C)} \times 2951.52 = - 14.2304 \text{ (deg C)} \]

**Conversion C to R**
Homework

• Plot $\theta$, $\delta$, $\sigma$ (x-axis) vs Altitude (y-axis from -1000 ft to 43,000 ft)