The Standard Atmosphere

SYST 460/560

Intro to Air Transportation System Engineering

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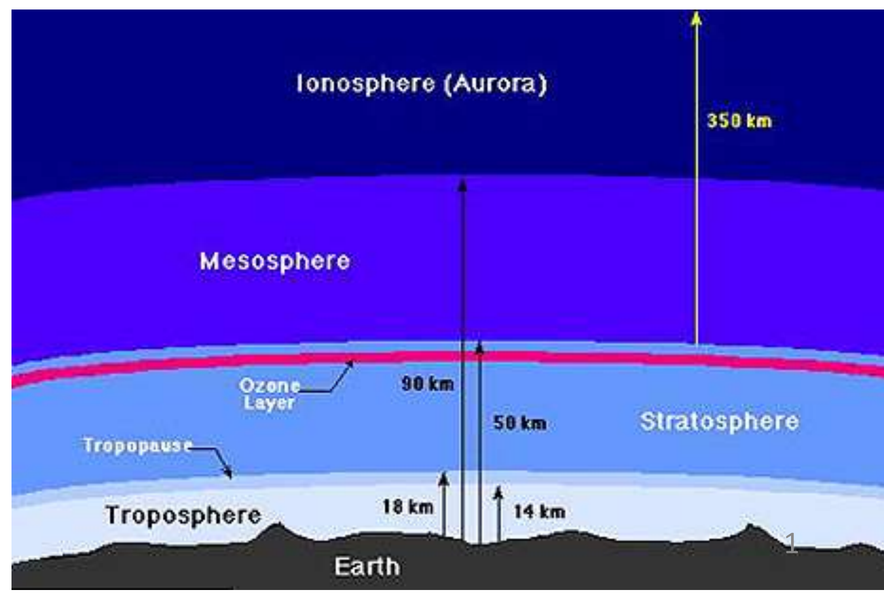


Motivation

- Aerodynamic and propulsive forces acting on aircraft depend on:
  - Local pressure (P)
  - Local temperature (T)
  - Local density (D)
  - Sonic velocity (a)

- How do P, T, D, a change as a function of altitude

Learning Objectives

Knowledge

- Hydrostatic equation
- Equation of state (for air as a perfect gas)
- Lapse Rate Equation
- Troposphere
- Stratosphere
- Sonic Velocity
- Standard Atmosphere

Skills

- Derive equations for (T, P, D, a) from basic equations for Troposphere and Stratosphere
- Calculate Ratios for Standard Atmosphere
- Calculate T, P, D, a for Standard Atmosphere

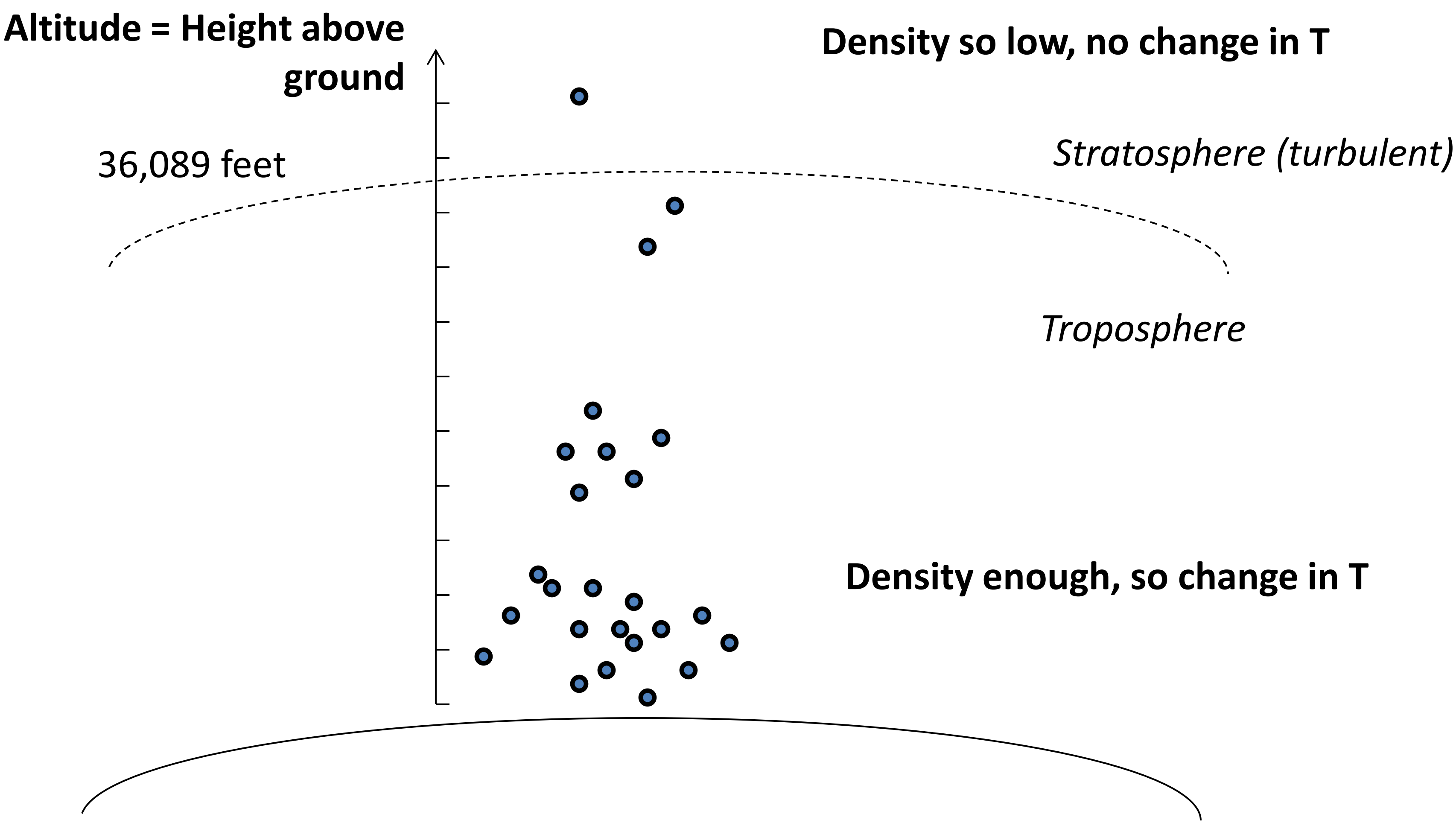
Composition of Atmosphere

- Air is treated as a perfect dry gas
  - 78% Nitrogen
  - 22% Oxygen
  - traces of other gases like Hydrogen, Carbon dioxide, …

Properties of Atmosphere

- Density = f (Altitude)
  - Density decreases as altitude increases
- Pressure = f (Altitude)
  - Pressure decreases as altitude increases
- Temperature = f (Altitude)
  - Temperature decreases as altitude increases
  - drops 1 deg C for every 1000 ft increase in altitude

Troposphere/Stratosphere



Standard Atmosphere

- Standard Atmosphere defines values for (P, T, D, a) as a function of altitude

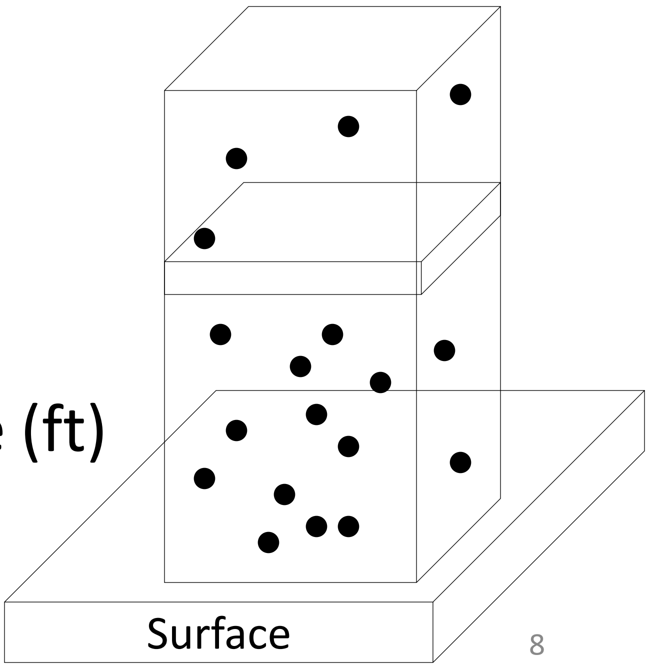
- Assumptions:
  – (1) atmosphere is static,
  – (2) rotates with the Earth

- Subsonic aircraft: surface to 45,000ft

Basic Equation #1 – Hydrostatic Equation

- The difference in pressure (dp) between two altitudes (dH), is equal to the weight (mass * gravitational constant).
- Relates pressure (p) and density (ρ) to height

- $dp = -\rho g \, dH$
  - p = barometric pressure (lb/ft^2)
  - ρ = density (sl/ft^3)
  - g = gravitational constant (ft/sec^2)
  - H = Height in Standard Atmosphere (ft)



Basic Equation #2 - Equation of State for Air (as a Perfect Gas)

- $P = \rho * R * T$

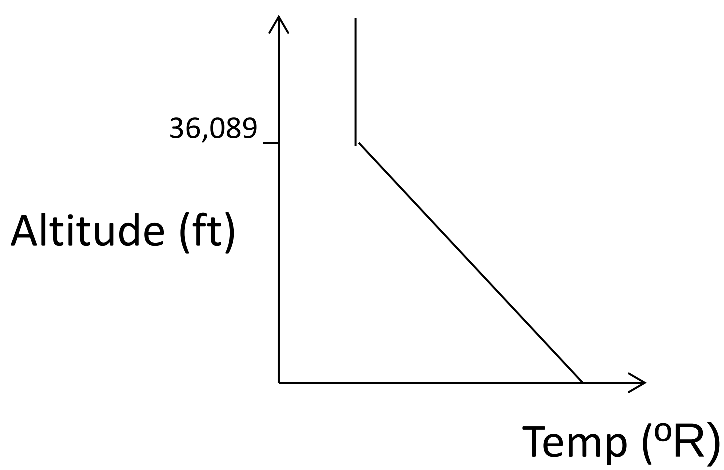
- Relates the pressure (p) and density (ρ) to the Temperature (T)
  - R = gas constant for air = 287.053 joules/kg-deg K = 1716.551 ft-lb/sl-degR
  - P_0 = standard sea-level pressure 101325 n/m^2 = 2116.22 lb/ft^2 = 29.9213 in Hg at T_0 = 518.67 deg R
  - P_0 = standard sea-level density 1,22500 kg/m^3 = 0.00237691 sl/ft^3

Basic Equation #3 – Temperature vs Altitude

- Troposphere (surface (-1000 ft) to H_T = 36,089 ft)
  - $T_T = T_0 + (L * H)$ $0 \leq H \leq H_T$ changes with altitude
- Stratosphere (greater than H_T = 36,089 ft)
  - Isothermal layer (i.e. constant temperature)
  - $T_S = T_0 + (L * H_T)$ $H > H_T$ does not change with altitude
- L = dT/dH = thermal lapse rate = -6.5 deg K per km = 5.5°F/1000 feet =
- T_0 = standard sea level temperature = 288.15 deg K = 518.67 deg R = 15 deg C = 59 deg F

Basic Equation #3 – Temperature vs Altitude

- Below 36,089 feet,
  - Ambient Temperature (^oR) = -3.566^o * (Altitude/1000)
- Above 36,089 feet
  - Ambient Temperature (^oR) = 389.988^o



Basic Equation #4 – Sonic Velocity

- Sonic Velocity = a

  a = SQRT (γ * R * T)

  – γ = ratio of specific heats for air = 1.4 (dimensionless)

  – a_0 = standard sea-level velocity 340.294 m/sec = 1116.45 ft/sec

Normalized Equations for Troposphere

- <u>Normalized Temperature Ratio</u>

$\Theta = T/T_0 = 1 + L\,(H/T_0)$

- <u>Normalized Pressure Ratio</u>

$\delta = P/P_0 = [1 + H/(T_0/L)\ ]$

- <u>Normalized Density Ratio</u>

$\sigma = \delta/\theta = [1 + H(T_0/L)]^{-(1+g/LR)}$

- <u>Normalized Sonic Velocity</u>

$\mu = a/a0 = SQRT\ (\theta)$

Normalized Equations for Stratosphere

- Normalized Temperature Ratio

$\Theta_S = T_s/T_0 = 1 + H_T\ (T_0/L)$

- Normalized Pressure Ratio

$\delta_S = \delta_T \exp\ [-\ (H – HT)/(RT_S/g)\ ]$

- Normalized Density Ratio

$\sigma_S = \delta_S / \theta_S = \delta_T/\theta_s\ \{\exp\ [-\ (H – HT)/(RT_S/g)\ ]\}$

- Normalized Sonic Velocity

$\mu_S = a/a_0 = SQRT\ (\theta_S)$

Numerical Constants

T_0/L = -145,442 ft

$-g/LR$ = 5.255913 (dimensionless)

T_S = 389.97 deg R

δ_T = 0.223359 (dimensionless)

RT_s/g = 20,805.7ft

Numerical Equations

Troposphere

$\theta = 1 – (6.8753 * 10\text{-}6)$ Pressure Altitude

$\delta = 1 – (6.88 * 10\text{-}6 * \text{Pressure Altitude})^{5.26}$

$\sigma = \delta/ \theta$

$\mu = a/a_0 = \text{SQRT}\ (\theta_S)$

Stratosphere

$\theta = \theta_s = T_S/T_0 = 0.751865$

$\delta = 0.22336\ \text{EXP}((36{,}089 – \text{Pressure Alt})/20{,}805.7)$

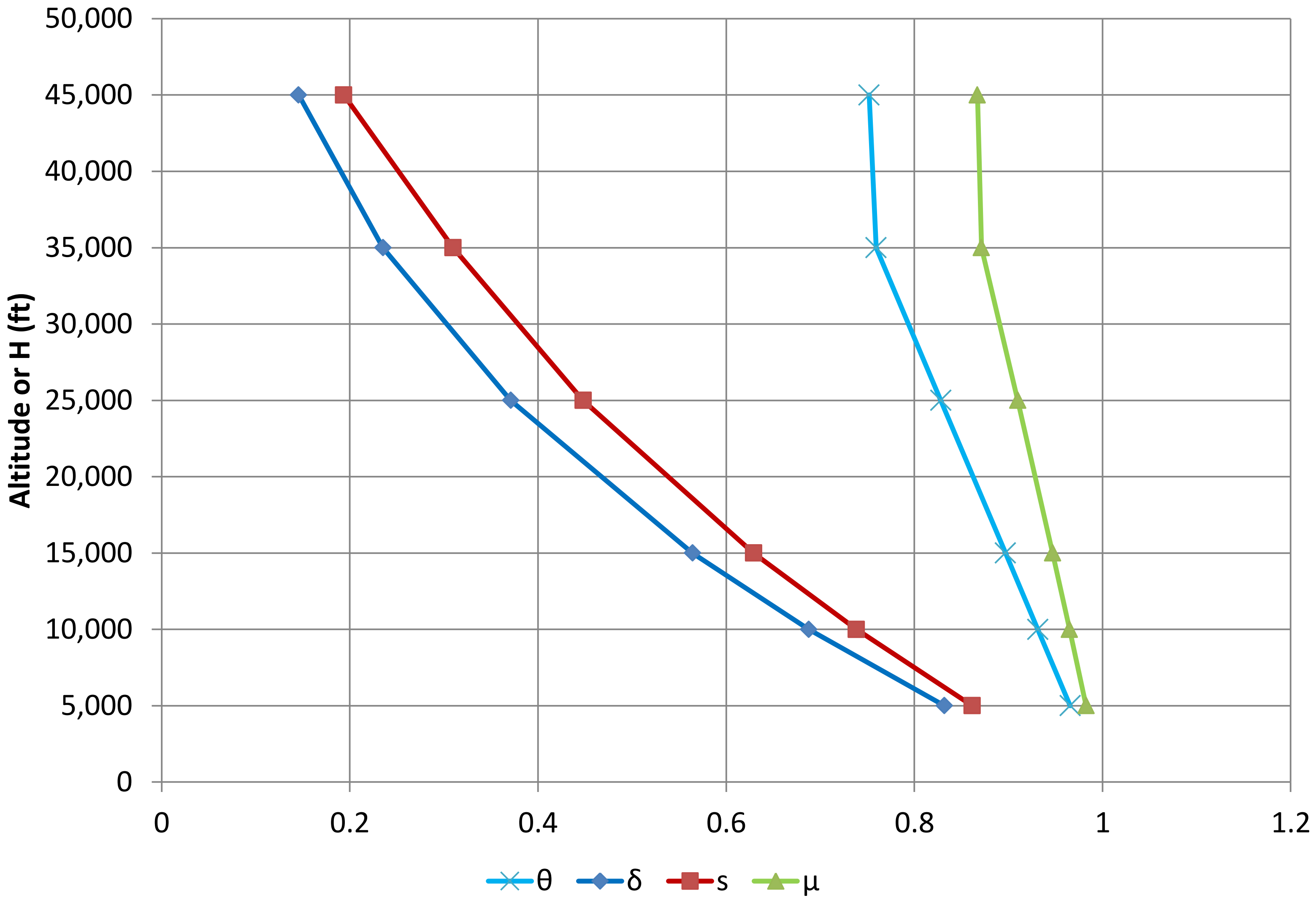
$\sigma = \delta/ \theta$

$\mu = a/a_0 = \text{SQRT}\ (\theta_S) = 0.867107$

Tabulated Values

Sea-level Properties				
H (ft)	T_0 (deg R)	P_0 (lb/ft2)	P_0 (sl/ft^3)	a_0 (ft/sec)
0	518.67	2116.22	0.00237691	1,116.45
	θ	δ	σ	μ
5,000	0.965622	0.832047	0.861669	0.982661
10,000	0.931244	0.687702	0.738447	0.965010
15,000	0.896866	0.564339	0.629235	0.947030
25,000	0.828110	0.371089	0.448116	0.910006
35,000	0.759354	0.235302	0.309872	0.871409
45,000	0.751865	0.145546	0.193580	0.867101

Graphed Values



Altitude	P	T	D
Altitude	Pressure	Temp.	Density -
(ft)	(in. Hg)	(F.)	slugs per cub
0	29.92	59.0	0.002378
1,000	28.86	55.4	0.002309
2,000	27.82	51.9	0.002242
3,000	26.82	48.3	0.002176
4,000	25.84	44.7	0.002112
5,000	24.89	41.2	0.002049
6,000	23.98	37.6	0.001988
7,000	23.09	34.0	0.001928
8,000	22.22	30.5	0.001869
9,000	21.38	26.9	0.001812
10,000	20.57	23.3	0.001756
11,000	19.79	19.8	0.001701
12,000	19.02	16.2	0.001648
13,000	18.29	12.6	0.001596
14,000	17.57	9.1	0.001545
15,000	16.88	5.5	0.001496
16,000	16.21	1.9	0.001448
17,000	15.56	-1.6	0.001401
18,000	14.94	-5.2	0.001355
19,000	14.33	-8.8	0.001310
20,000	13.74	-12.3	0.001267
25,000	11.10	-30.15	
30,000	8.89	-47.98	
35,000	7.04	-68.72	
40,000	5.54	-69.70	
45,000	4.35	-69.70	
50,000	3.43	-69.70	
55,000	2.69	-69.70	
60,000	2.12	-69.70	
65,000	1.67	-69.70	
70,000	1.31	-69.70	
75,000	1.03	-69.70	
80,000	0.81	-69.70	
85,000	0.64	-64.80	
90,000	0.50	-56.57	
95,000	0.40	-48.34	
100,000	0.32	-40.11	

Test Yourself

Q: Compute the Standard Atmosphere Temperature at H=35,000 ft

T_0

Θ for 35,000 ft

A:

1. $\theta = T/T_o$
2. $T = \theta * T_o = 0.759354 * 518.67$ (deg R)
3. T = 3886.9 (deg R)
4. T = 1 (degR)/-272.594444(deg C) * 2951.52 = - 14.2304 (deg C)

Conversion C to R

Homework

- Plot θ, δ, σ (x-axis) vs Altitude (y-axis from -1000 ft to 43,000 ft)