Aircraft Dynamics

**Pitch Dynamics**

Notes

- Transfer function \( \frac{1}{s+1} \) maps the input to the output for the following differential equation:
  \[ \dot{x} + x = f(t) \] (\( x \) is output, \( f(t) \))
- Based on input, we are solving the equation: \( \dot{x} + x = \theta_{cmd} \)
- The solution is \( x(t) = \theta_{cmd}(1-e^{-t}) \), which represents a delay between the input and output commands.

**Aircraft Dynamics**

Note: For small \( \theta \), \( \sin \theta \approx \theta \) (\( \theta \) must be in radians).

Reason: \( \sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots \approx \theta \)
**Control system framework**

![Control system diagram]

Proportional Control: \( G_c = K_p \)

**Closed Loop on Altitude**

![Closed loop diagram on altitude]

**Proportional + Derivative Control**

Proportional + Derivative Control: \( G_c = K_p (1 + K_d s) \)

Goal: Maintain altitude at some target value.
The control law is equivalent to the following:
action = \( K_p \left[ (z_{\text{target}} - z_{\text{actual}}) + K_d \left( \frac{dz_{\text{target}}}{dt} - \frac{dz_{\text{actual}}}{dt} \right) \right] \)

\[ = K_p \left[ (z_{\text{target}} - z_{\text{actual}}) - K_d \frac{dz_{\text{actual}}}{dt} \right] \]

That is, you take positive corrective action (pitch command > 0) if you are (a) too low \((z_{\text{target}} - z_{\text{actual}} > 0)\) or (b) if you are descending, or (c) some combination of the two.

**Closed Loop w/ Proportional + Derivative Control**

Summary of outcomes:
- Example of flight dynamics and implementation in Simulink
- Understanding of two different control mechanisms
- Implementation in MATLAB and numerical experiments showing differences between control laws