Aircraft Dynamics for Surface Operations and Takeoff

The objective of this section is to derive the equatins of motion for an aircraft on the airport surface (i.e. runway, taxiway, ramp).

The equations of motion are derived form Newtons 2nd law F = ma. Where a = dV/dt, and V = ds/dt. V is velocity, and s is distance traversed.

Forces

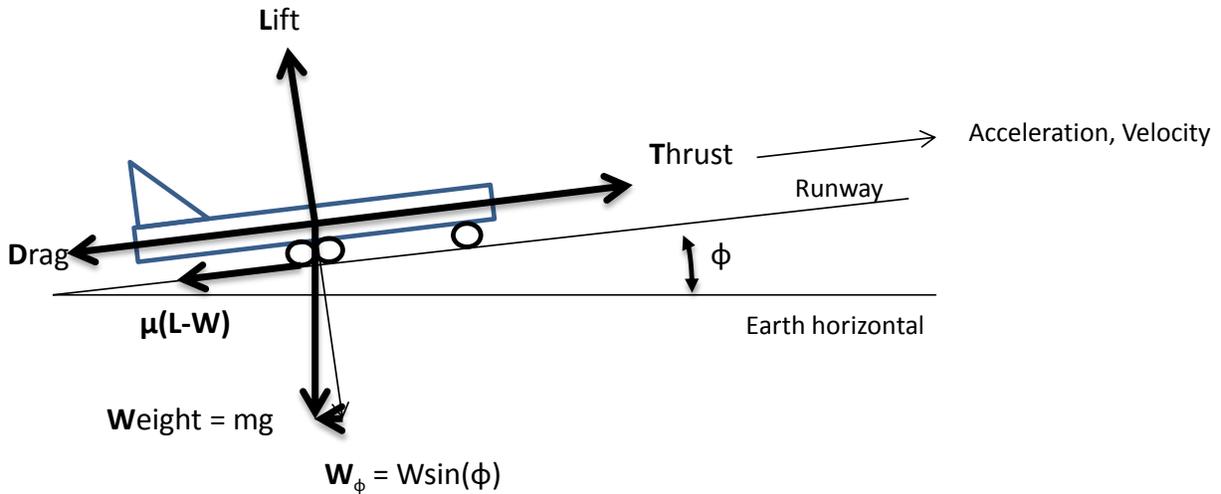The forces and acceleration are shown in the Figure below.

The Lift force is the total lift force generated by the aerodynamic surfaces as the aircraft moves. Lift is perpendicular to the direction of motion of the aircraft. Note that the Lift force is subject to ground effects and is not calculated directly from the Lift Coefficient (C_L) used for airborne dynamics.

The Drag for is also generated by the aerodynamics. Drag is parallel to the direction of motion of the aircraft Drag is also subject to ground effect.

The Rolling Friction force is the resistance to motion of the tires and the tire axles on the aircraft. Rolling Friction is parallel to the direction of motion of the aircraft Drag is also subject to ground effect. Note, this rolling friction is not the same as the friction between tires and pavement used in braking (where slippage between the tire and pavement occurs).

The Weight of the aircraft acts perpendicular to the Earth, not to the local surface (i.e. runway surface). The Weight exerts a negative force in the direction of motion of the aircraft defined by $W_\phi = W \sin\phi$. For low angles, $\sin\phi$ can be replaced by ϕ in radiands to simplfy the calculation.

Φ is the angle between the horizontal (i.e. Earth) and the runway surface. Φ is computed by the difference in elevation between runway ends (i.e. Touchdown Zone Elevation - TDZE) and the length of the runway.

Lift

Acceleration, Velocity

Thrust

Runway

Drag

φ

μ(L-W)

Earth horizontal

Weight = mg

$W_\phi = W\sin(\phi)$

Equations of Motion

The sum of the forces in direction of motion of the aircraft on the runway is:

$$\Sigma F = T - D - \mu(L-w) - W\sin\phi$$

Substituting in Newtons' 2nd law $\Sigma F = ma$

$$mdV/dt = T - D - \mu(L-W) - W\sin\phi$$

$$dv/dt = (T - D - \mu(L-W) - mg\sin\phi)/m$$

$$dv/dt = (T - D - \mu(L-W))/m - g\sin\phi$$

The Thrust required to maintain a constant speed (i.e. $dv/dt = 0$) is:

$$T = (D + \mu(L-W))/m + g\sin\phi$$

That is the Thrust is required to overcome Drag, the Rolling Friction (at the specified velocity) and the component of the Weight in the direction of motion of the vehicle.

The Thrust required to accelerate at a fixed arte (i.e. dv/dt = constant) is:

$$T = D + \mu(L-W) + g\sin\phi + mdV/dt$$

That is the Thrust is required to overcome Drag, the Rolling Friction (at the specified velocity), the component of the Weight in the direction of motion of the vehicle, and inertia (i.e. mdV/dt).

Simulation

For the purpose of discrete time simulations it is useful to describe the equations as difference equations.

$$dV/dt = (V_n - V_{n-1})/(t_n - t_{n-1})$$

where:

t = time (s)

n = current value

$n - 1$ = previous value

The equation for the acceleration (dV/dt) can be written as follows;

$$dV/dt = (V_n - V_{n-1})/(t_n - t_{n-1}) = T - D - \mu(L-W))/m - g\sin\phi$$

$$V_n = (T - D - \mu(L-W))/m - g\sin\phi) (t_n - t_{n-1}) + V_{n-1} +$$

That is the current Velocity (V_n) is equal to the acceleration over the last time period divided by the time period ($t_n - t_{n-1}$), plus the previous value of the velocity.

Likewise the distance can be computed by defining the Velocity (V) as the difference in distance covered over the last time period.

$$V_n = (S_n - S_{n-1})/(t_n - t_{n-1})$$

So distance can be computed as follows:

$$S_n = (t_n-t_{n-1}) [(T - D - \mu(L-W))/m - g\sin\phi) (t_n - t_{n-1})+ V_{n-1}] + s_{n-1}$$

$$S_n = V_{n-1} (t_n-t_{n-1}) + (T - D - \mu(L-W))/m - g\sin\phi) (t_n-t_{n-1})^2 + s_{n-1}$$

That is the current distance is the acceleration in the last time period times the time period squared, plus the velocity from the last time period times the time period, plus the previous distance.

Homework:

1) Draw the free-body diagram of the aircraft on the runway surface. Include all forces and angles. Include the direction of motion.
2) Derive the equations of motion of the vehicle in terms of the acceleration of the vehicle in the direction of motion along the runway.
3) Using the equation in 2,

    a. Define how much Thrust is required to maintain a *constant speed* on the taxiway. Explain each term.

    b. Define how much Thrust is required to maintain a *constant acceleration* on the runway. Explain each term.

    c. Define how much additional thrust is required to maintain a constant *acceleration* when going from a flat runway ($\phi = 0$) to runway with a slope of 3 degrees ($\phi = 3$ degrees)

4) Using the facts that dV/dt = ΔVelocity/Time and velocity = ΔDistance/Time derive the equations of motion for Velocity (V_n) and Distance (s_n) as difference equations that can be used in a discrete time simulation.